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M8H Ch2a Operations with Fractions 2025

1. When adding or subtracting two fractions, why do they need to have a common denominator first?
   Explain: It is because although the two fractions might have the same number, their value will be different since they might have different denominators.
   Just like $2x-2y$ is not $2x-2x$, $\frac{1}{x}-\frac{1}{y}$ is not $\frac{1}{x}-\frac{1}{x}$.
2. When multiplying or dividing fractions, do you need to have a common denominator first? Why or why not? Explain?
   You don't need a common denominator when dividing or multiplying fractions, you only need it for addition and subtraction.
3. When dividing by a fraction, why do we flip the fraction and change the division operation to a multiplication operation? Explain:
   It is because dividing a number(x) by another number(y) is the same as multiplying it (x) by the other number(y)'s reciprocal($\frac{1}{y}$). $x \div y = x \times \frac{1}{y}$
4. When multiplying fractions, what does it mean to "Cancel Out" common factors? Explain:
   It means using the GCD of a fraction's numerator and another fraction's denominator to reduce them to the lowest term.
   For example the GCD of $\frac{7}{2} \times \frac{14}{3}$ is 2, we divide 2 and 4 by 2 $\rightarrow \frac{7}{1} \times \frac{7}{3} = \frac{14}{3}$
5. When multiplying fractions, can you "cancel" out a common factor if both terms are in the numerator? Why or Why not? Explain.
   No, you can only cancel out a common factor between the denominator and numerator. Cancelling the two numerators will change the fraction.
   For example, $3 \times 3 = 9$ or $\frac{3}{1} \times \frac{3}{1} = 9$ If you cancel the numerators it won't become $\frac{1}{1} \cdot \frac{1}{1} = 1$.
6. What does it mean to find the reciprocal? What is the reciprocal of $\frac{a}{b}$?
   Finding the reciprical means finding the opposite of a number.
   The reciprical of $\frac{a}{b}$ is $\frac{b}{a}$.
7. What happens when you take a number to the power of -1? Suppose you have a fraction to the power of -1, what would it become?
   When a number has a power of -1, it is basically $\frac{1}{n^1} \rightarrow \frac{1}{n}$
   When I have a fraction to the power of -1, the numerator and denominator swaps.
8. Indicate the GCF between the numerator and denominator. Then reduce the fractions to lowest terms

a) $\frac{8}{12}$ GCF=4 $\frac{8\div4}{12\div4}=\frac{2}{3}$	b) $\frac{15}{25}$ GCF=5 $\frac{15\div5}{25\div5}=\frac{3}{5}$	c) $\frac{24}{36}$ GCF=12 $\frac{24\div12}{36\div12}=\frac{2}{3}$	d) $\frac{36}{63}$ GCF=9 $\frac{36\div9}{63\div9}=\frac{4}{7}$	e) $\frac{56}{49}$ GCF=7 $\frac{56\div7}{49\div7}=\frac{8}{7}$	f) $\frac{48}{84}$ GCF=12 $\frac{48\div12}{84\div12}=\frac{4}{7}$
g) $\frac{65}{117}$ GCF=13 $\frac{65\div13}{117\div13}=\frac{5}{9}$	h) $\frac{150}{105}$ GCF=15 $\frac{150\div15}{105\div15}=\frac{10}{7}$	i) $\frac{224}{36}$ GCF=4 $\frac{224\div4}{36\div4}=\frac{56}{9}$	j) $\frac{24}{96}$ GCF=24 $\frac{24\div24}{96\div24}=\frac{1}{4}$	k) $\frac{81}{135}$ GCF=27 $\frac{81\div27}{135\div27}=\frac{3}{5}$	L) $\frac{720}{1080}$ GCF=360 $\frac{720\div360}{1080\div360}=\frac{2}{3}$
M) $\frac{a^5b}{a^4b^3}$ GCF=a^4b $\frac{a^5b\div a^4b}{a^4b^3\div a^4b}=\frac{a}{b^2}$	N) $\frac{8!}{2^9}$ GCF=2^7 $\frac{2^7\cdot3^2\cdot5\cdot7\div2^7}{2^9\div2^7}=\frac{315}{4}$	O) $\frac{a^2b^3c}{ab^5c^2}$ GCF=ab^3c $\frac{a^2b^3c\div ab^3c}{ab^5c^2\div ab^3c}=\frac{a}{b^2c}$	P) $\frac{3a+3b}{5a+5b}$ GCF=$a+b$ $=\frac{3\div3}{5\div5}=\frac{6}{10}=\frac{3}{5}$	Q) $\frac{a^2-b^2}{2a+2b}$ GCF=$a+b$ $=\frac{(a+b)(a-b)}{2(a+b)}=\frac{a-b}{2}$	R) $\frac{a-b}{b-a}$, $[a\neq b]$ $=-1$

9. Evaluate the following by adding or subtracting fractions

a) $\frac{5}{9}+\frac{40}{72}$ $=\frac{40}{72}+\frac{40}{72}$ $=\frac{80}{72}=\frac{10}{9}$	b) $\frac{3}{18}+\frac{25}{30}$ $=\frac{15}{90}+\frac{75}{90}$ $=\frac{90}{90}=1$	c) $\frac{10}{21}+\frac{22}{28}$ $=\frac{40}{84}+\frac{66}{84}$ $=\frac{106}{84}=\frac{53}{42}$
d) $1\frac{2}{5}+3\frac{2}{20}$ $=1\frac{8}{20}+3\frac{2}{20}$ $=4\frac{10}{20}=4\frac{1}{2}=\frac{9}{2}$	e) $1\frac{5}{17}-\frac{38}{51}$ $=1\frac{15}{51}-\frac{38}{51}=\frac{66}{51}-\frac{38}{51}$ $=\frac{28}{51}$	f) $\frac{34}{39}-\frac{55}{78}$ $=\frac{68}{78}-\frac{55}{78}$ $=\frac{13}{78}=\frac{1}{6}$
g) $2\frac{1}{5}+3\frac{2}{4}-2\frac{7}{3}$ $=\frac{11}{5}+\frac{14}{4}-\frac{13}{3}=\frac{132}{60}+\frac{210}{60}-\frac{260}{60}$ $=\frac{82}{60}=\frac{41}{30}$	h) $-3\frac{1}{4}+3\frac{2}{3}+1\frac{1}{2}$ $=-\frac{13}{4}+\frac{11}{3}+\frac{3}{2}=-\frac{39}{12}+\frac{44}{12}+\frac{18}{12}$ $=\frac{23}{12}$	i) $4\frac{7}{8}-2\frac{9}{10}-2\frac{13}{20}$ $=\frac{39}{8}-\frac{29}{10}-\frac{53}{20}=\frac{195}{40}-\frac{116}{40}-\frac{106}{40}$ $=\frac{27}{40}$
j) $\frac{15}{14}\times\frac{21}{36}-\frac{2}{3}$ $=\frac{5}{8}-\frac{2}{3}=\frac{15}{24}-\frac{16}{24}$ $=-\frac{1}{24}$	k) $\frac{35}{36}\div\frac{49}{48}-\frac{2}{7}$ $=\frac{35}{36}\times\frac{48}{49}-\frac{2}{7}=\frac{20}{21}-\frac{6}{21}$ $=\frac{14}{21}=\frac{2}{3}$	L) $\frac{1}{3}-2\frac{2}{3}\times5\frac{1}{4}$ $=\frac{1}{3}-\frac{8}{3}\times\frac{21}{4}=\frac{1}{3}-14$ $=13\frac{2}{3}=\frac{41}{3}$
m) $\frac{5}{2}\times\frac{4}{15}+\frac{1}{2}$ $=\frac{2}{3}+\frac{1}{2}=\frac{4}{6}+\frac{3}{6}$ $=\frac{7}{6}$	n) $\frac{2}{3}+\frac{6}{7}\div\frac{18}{14}-\frac{1}{2}$ $=\frac{2}{3}+\frac{6}{7}\times\frac{14}{18}-\frac{1}{2}$ $=\frac{2}{3}+\frac{2}{3}-\frac{1}{2}=\frac{4}{6}+\frac{4}{6}-\frac{3}{6}$ $=\frac{5}{6}$	O) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ $=\frac{30}{60}+\frac{20}{60}+\frac{15}{60}+\frac{12}{60}+\frac{10}{60}$ $=\frac{87}{60}=\frac{29}{20}$

10. Evaluate the following by Multiply or Divide the following.

a) $\frac{12}{18}\times\frac{27}{15}$ $=\frac{6}{5}$	b) $\frac{25}{24}\times\frac{16}{15}$ $=\frac{10}{9}$	c) $\frac{10}{12}\div\frac{1}{3}$ $=\frac{10}{12}\times\frac{3}{1}$ $=\frac{5}{2}$

d) $\frac{6}{7} \div \frac{3}{4}$

$= \frac{6^2}{7} \times \frac{4}{3_1}$

$= \frac{8}{7}$

e) $4\frac{2}{3} \div 1\frac{2}{7}$

$= \frac{14}{3} \div \frac{9}{7} = \frac{14}{3} \times \frac{7}{9}$

$= \frac{98}{27}$

f) $\frac{16}{21} \div \frac{24}{35}$

$= \frac{16^4}{21_3} \times \frac{35^5}{24_3}$

$= \frac{10}{9}$

g) $6\frac{3}{4} \div \frac{3}{16}$

$= \frac{27^9}{4_1} \times \frac{16^4}{3_1}$

$= 36$

h) $3\frac{1}{5} \times 6\frac{1}{4} \div 13\frac{1}{3}$

$= \frac{16}{5} \times \frac{25}{4} \div \frac{40}{3} = \frac{16^{21}}{5_1} \times \frac{25^{5-1}}{4_1} \times \frac{3}{40_{8-1}}$

$= \frac{3}{2}$

i) $\frac{26}{24_6} \times \frac{12^2}{14_1} \times \frac{56^4}{65_1}$

$= \frac{52}{65}$

j) $\frac{25^5}{21_{9-1}} \times \frac{54^{3-1}}{11_1} \times \frac{28^4}{15_3} \times \frac{22^2}{18_1}$

$= \frac{40}{3}$

k) $\frac{14^2}{27_9} \times \frac{18^{6-2}}{40_5} \times \frac{36^4}{15_5} \times \frac{32^4}{21_{3-1}}$

$= \frac{64}{75}$

L) $\frac{16}{21} \times \frac{30}{24} \times \frac{27}{18} \div \frac{9}{14}$

$= \frac{16^2}{21_{3-1}} \times \frac{30^5}{24_3} \times \frac{27^{3-1}}{18_3} \times \frac{14^2}{9_1}$

$= \frac{20}{9}$

m) $\frac{64}{9} \times \frac{27}{32} \times \frac{24}{45} \div \frac{48}{18}$

$= \frac{64^{32-2}}{9_1} \times \frac{27^3}{32_{4-2-1}} \times \frac{24^{3-1}}{45_5} \times \frac{18^{2-1}}{48_{16-1}}$

$= \frac{6}{5}$

n) $16 \times \frac{22}{12} \times \frac{27^{3-1}}{25} \times \frac{15^3}{18_{2-1}}$

$= \frac{132}{5}$

O) $\frac{1}{3} \div \frac{4}{27} \div \frac{16}{21}$

$= \frac{1}{3_1} \times \frac{27}{4} \times \frac{21^7}{16}$

$= \frac{189}{64}$

$1\frac{2}{3} \div 2\frac{3}{6} \div 6\frac{1}{4}$

$= \frac{5}{3} \div \frac{15}{6} \div \frac{25}{4}$

$= \frac{5^1}{3_1} \times \frac{6^2}{15_3} \times \frac{4}{25}$

$= \frac{8}{15}$

$\frac{2}{3} \div 2\frac{3}{4} \div 7\frac{1}{3}$

$= \frac{2}{3} \div \frac{11}{4} \div \frac{22}{3}$

$= \frac{2^1}{3_1} \times \frac{4}{11} \times \frac{3^1}{22_{11}}$

$= \frac{4}{121}$

$7\frac{3}{4} \div 5\frac{1}{3} \div 1\frac{1}{2}$

$= \frac{31}{4} \div \frac{16}{3} \div \frac{3}{2}$

$= \frac{31}{4} \times \frac{3^1}{16_2} \times \frac{2^1}{31}$

$= \frac{31}{32}$

11. Simplify each of the following composite fractions. Reduce your answer to lowest form

a) $\frac{\left[\frac{2}{3} + \frac{3}{4}\right] \times 60}{\left[\frac{4}{5} - \frac{1}{6}\right] \times 60} = \frac{40 + 45}{48 - 10}$

$= \frac{85}{38}$

b) $\frac{\frac{7}{8} + \frac{3}{5}}{\frac{1}{2} - \frac{3}{6}} =$ undefined

$\frac{3}{6} - \frac{3}{6} = 0$

a fraction cannot have a denominator of 0.

c) $\frac{\frac{3}{4} \times \frac{1}{2} - \frac{2}{7}}{\frac{5}{3} + \frac{1}{2}} = \frac{\frac{3}{8} - \frac{2}{7}}{\frac{10}{6} + \frac{3}{6}} = \frac{\frac{5}{56}}{\frac{13}{6}}$

$= \frac{5}{56} \div \frac{13}{6} = \frac{5}{56_{28}} \times \frac{6^3}{13} = \frac{15}{364}$

d) $\dfrac{\frac{2}{3}\times\frac{6^2}{5}-\frac{1}{10}}{\frac{1}{5}+\frac{5}{2}} = \dfrac{\left[\frac{4}{5}-\frac{1}{10}\right]\times 10}{\left[\frac{1}{5}+\frac{5}{2}\right]\times 10}$

$= \frac{8-1}{2+10} = \frac{7}{12}$

e) $\dfrac{\left(2\frac{1}{2}+3\frac{3}{4}\right)\times\frac{8}{50}}{\frac{2}{5}} = \dfrac{\left[\frac{10}{4}+\frac{15}{4}\right]\times\frac{4}{25}}{\frac{2}{5}}$

$= \frac{\frac{25}{4_1}\times\frac{4^1}{25_1}}{\frac{2}{5}} = \frac{1}{\frac{2}{5}}$

$= \frac{5}{2}$

f) $\dfrac{\frac{1}{2}+\left(\frac{3}{4}\div\frac{1}{2}\right)}{\left(\frac{1}{6}+\frac{2}{3}\right)\div\frac{3}{5}} = \dfrac{\frac{1}{2}+\frac{3}{2}}{\frac{5}{6}\times\frac{5}{3}}$

$= \frac{2}{\frac{25}{18}} = 2\times\frac{18}{25}$

$= \frac{36}{25}$

12. Simplify each of the fractions to the given exponent. Reduce your answer to lowest form

a) $\left(\frac{2}{3}\right)^{-1}$

$= \frac{3}{2}$

b) $\left(\frac{4}{5}\right)^{-1}$

$= \frac{5}{4}$

c) $\left(\frac{5}{4}\right)^{-2} = \frac{4^2}{5^2}$

$= \frac{16}{25}$

d) $\left(\frac{a^2}{b^3}\right)^{-1} = \frac{b^3}{a^2}$

e) $\left(\frac{a^2c^3}{b^2d}\right)^{-2} = \frac{b^4d^2}{a^4c^6}$

f) $\dfrac{\frac{2}{4}}{7} = \frac{14}{4} = \frac{7}{2}$

g) $\dfrac{\frac{1}{1}}{\frac{1}{\left(\frac{a}{b}\right)}} = \frac{1}{\frac{1}{\frac{b}{a}}} = \frac{1}{\frac{a}{b}} = \frac{b}{a}$

h) $\left(\dfrac{1}{\frac{1}{\frac{1}{\frac{a}{b}}}}\right)^{-2} = \left(\frac{b}{a}\right)^{-2} = \frac{a^2}{b^2}$

i) $\dfrac{2}{\frac{4}{7}+\frac{1}{\frac{2}{3}+\frac{1}{2}}} = \dfrac{2}{\frac{4}{7}+\frac{1}{\frac{7}{6}}} = \dfrac{2}{\frac{4}{7}+\frac{6}{7}}$

$= \frac{2}{\frac{10}{7}} = \frac{14}{10} = \frac{7}{5}$

Part 2 Applications of Fractions: For each of the following questions below, write out the mathematical operations required to solve the problem. Then solve the problem.

13. Albert spent 1/5 of his savings on a new computer. Later that week, he spent ½ of the rest on another desktop computer. What fraction of his original savings did he have left?

$1-\frac{1}{5}-\frac{1}{2}$

$= 1-\frac{2}{10}-\frac{5}{10}$

$= \frac{3}{10}$

He had $\boxed{\frac{3}{10}}$ of his original savings left.

14. Jenny has a purse of coins. 3/7 of the coins are pennies. 1/8 of the other coins are quarters. If there are 15quarters, how many coins pennies does she have?

$15\times 8\times\frac{3}{4}$ → to get the amount of other coins

→ the amount of other coins is $\frac{4}{7}$ of the total amount of all the coins

$\frac{4}{7}\times\frac{3}{4} = \frac{3}{7}$ = number of pennies

$=15\times 6$

$=90$

She has $\boxed{90}$ pennies.

15. Shirley bought a bunch of donuts and left it in the common room. Mike arrives and eats 1/3 of all the donuts. Then Eva arrives and eats ¼ of what remained. If there was 3 donuts left, how many donuts did Shirley buy?

$3 \div \frac{3}{4} \div \frac{2}{3}$

$= 3 \times \frac{4}{3} \times \frac{3}{2}$

$= 6$

Shirley bought [6] donuts.

16. Rick has a string 60 meters. He cuts the string into pieces that are 2/3 of a meter long. How many pieces will he have?

$60 \div \frac{2}{3}$

$= 60 \times \frac{3}{2}$

$= 90$

He will have [90] pieces.

17. Sally and her friends bought three pizzas, with eight slices each. One of the boys ate 3 slices and went home. They now need to split the rest of the pizza amongst six people. What fraction of a pizza does each person get?

$\left(\frac{24}{8} - \frac{3}{8}\right) \div 6$

$= \frac{21}{8} \times \frac{1}{6}$

$= \frac{7}{16}$

Each person get [$\frac{7}{16}$] of a pizza.

18. Michael has a piece of tape $7\frac{4}{5}$ units long. If he cuts it into pieces each $\frac{3}{5}$ of a unit long, how many pieces will he have?

$7\frac{4}{5} \div \frac{3}{5}$

$= \frac{39}{5} \times \frac{5}{3}$

$= 13$

He will have [13] pieces.

19. A cookie recipe requires $1\frac{2}{3}$ cups of flour to make 10 cookies. If Sarah has 18cups of flour, how many cookies can she make?

$18 \div 1\frac{2}{3} \times 10$

$= 18 \times \frac{3}{5} \times 10$ $= 18 \times 6 = 108$

She can make [108] cookies.

20. An earthquake in Japan was $2\frac{1}{2}$ times greater than the earthquake in Italy. Another earthquake in Chile was $11\frac{2}{3}$ greater than the earthquake in Italy. How many times greater was the earthquake in Chile compared to Japan? $11\frac{2}{3} \div 2\frac{1}{2}$

$= \frac{35}{3} \div \frac{5}{2}$ $= \frac{35}{3} \times \frac{2}{5}$

$= \frac{14}{3}$ $= 4\frac{2}{3}$

The earthquake in Chile was [$4\frac{2}{3}$] times greater than the earthquake in Japan.

21. A song on your iPhone is $3\frac{3}{4}$ minutes long. If you have $9\frac{2}{3}$ minutes, how many times can you listen to the song?

$9\frac{2}{3} \div 3\frac{3}{4}$ $= \frac{29}{3} \div \frac{15}{4}$

$= \frac{29}{3} \times \frac{4}{15}$ $= \frac{116}{45}$

$= 2\frac{26}{45}$

I can listen to the song [2] times.

22. Jason needs $2\frac{1}{2}$ tanks of gas to drive from Vancouver to Portland. Each tank of gas will cost him $\$\frac{98}{3}$.

How much will it cost him to drive from Vancouver to Portland?

$2\frac{1}{2} \times \frac{98}{3} = \frac{5}{2_1} \times \frac{98}{3}$

$= \frac{245}{3}$

It will cost him $\boxed{\$\frac{245}{3}}$ to drive from Vancouver to Portland.

23. The length of a box is increased by $1\frac{2}{3}$ times its original length and the width is increased by $2\frac{1}{5}$ times its original width. If the original area of the box is $300m^2$, then what is the area of the new box?

$300 \times 1\frac{2}{3} \times 2\frac{1}{5}$

$= 300 \times \frac{5}{3} \times \frac{11}{5}$

$= 1100$

The area of the new box is $\boxed{1100m^2}$.

24. Challenge: Sharon has some money in her pocket. Her friend Wendy has $1\frac{1}{2}$ times as much as Sharon. Another friend Chelsea has $1\frac{2}{3}$ times as much money as Wendy. Altogether they have \$200. How much money does Sharon have?

$S + \frac{3}{2}S + \frac{3}{2}S \cdot \frac{5}{3} = 200$ S = amount of Sharon's money

$S + \frac{3}{2}S = 200 \rightarrow 5x = 200 \rightarrow x = 40$

Sharon has $\boxed{40 \text{ dollars}}$.

25. Larry watched one television program for $\frac{1}{3}$ of an hour and then watched another program for 15 min. For what fraction of an hour did Larry watch television?

$\frac{1}{3} + \frac{1}{4}$

$= \frac{4}{12} + \frac{3}{12}$

$= \frac{7}{12}$

Larry watched television for $\boxed{\frac{7}{12}}$ of an hour.

26. Keon baked a wild blueberry upside-down cobbler. Shawnie ate $\frac{1}{6}$ of the cobbler. Iris ate $\frac{1}{5}$ of what was left. Chan ate $\frac{1}{4}$ of what was left after that. Cami ate $\frac{1}{3}$ of what was left after that. Demi ate $\frac{1}{2}$ of what was left after that. How much of the original cobbler remained?

$1 - \frac{1}{6} - \frac{1}{6} - \frac{1}{6} - \frac{1}{6} - \frac{1}{6}$

$= \frac{1}{6}$

$\boxed{\frac{1}{6}}$ of the original cobbler remained.

27. Challenge: Amy, Betty, and Graham ran for Student Council president. Amy won with $\frac{9}{20}$ of the votes, Betty got $\frac{2}{5}$, and Graham got $\frac{3}{20}$. If 20 people had switched their vote from Graham to Betty, then Betty would have ended up with 1 more vote than Amy. How many people voted?

x = number of people voted

$\frac{8}{20}x + 20 = \frac{9}{20}x + 1$

$\frac{9}{20}x + \frac{8}{20}x + \frac{3}{20}x = x$

(Amy + Betty + Graham = total votes)

$19 = \frac{1}{20}x$

$19 \times 20 = x$

$380 = x$

$\boxed{380}$ people voted.

Name: Francis Date:

Math 8H HW CH2 Lesson 2 Solving Equations with Simple Operations

1. Solve the following equations for "x" . If the answer is a fraction, simplify to lowest terms.

a) $3x-5=13$

$3x=13+5$

$3x=18$

$x=6$

b) $\frac{5n}{6}=12$

$5n=72$

$n=\frac{72}{5}$

c) $8+5x=12$

$5x=12-8$

$5x=4$

$x=\frac{4}{5}$

d) $-6x+4=-8$

$-6x=-8-4$

$-6x=-12$

$x=2$

e) $12=\frac{-4x}{3}+2$

$10=\frac{-4x}{3}$

$30=-4x$

$x=-\frac{4}{30}$

$x=-\frac{2}{15}$

f) $13=-3x-2$

$13+2=-3x$

$15=-3x$

$x=-5$

g) $\frac{2x}{5}-4=12$

$\frac{2x}{5}=12+4$

$\frac{2x}{5}=16$

$2x=80$

$x=40$

h) $\frac{3}{x}=\frac{1}{12}$

$\frac{3}{x}\times\frac{1}{3}=\frac{1}{12}\times\frac{1}{3}$

$\frac{1}{x}=\frac{1}{36}$

$x=36$

i) $9x-0.5=3x$

$9x-3x=0.5$

$6x=\frac{1}{2}$

$x=\frac{1}{12}$

j) $\frac{30x}{4}=\frac{20}{7}$

$\frac{30x}{4}\times 28=\frac{20}{1}\times 28$

$210x=80$

$x=\frac{80}{210}$

$x=\frac{8}{21}$

k) $x+(x+1)+(x+2)=-75$

$3x+3=-75$

$3x=-78$

$x=-26$

L) $3-\frac{2}{x}=5$

$-\frac{2}{x}=5-3$

$-\frac{2}{x}=2$

m) $\frac{3x}{8}=x+1$

$\frac{3x}{8}-x=1$

$3x-8x=8$

$-5x=8$

$x=-\frac{8}{5}$

n) $4x+12=8x-5$

$4x-8x=-5-12$

$-4x=-17$

$x=\frac{17}{4}$

o) $4x+2-9x=8+7x-4$

$4x-9x-7x=8-4-2$

$-12x=2$

$x=-\frac{1}{6}$

p) $x+(x+1)+(x+2)=-75$

$3x+3=-75$

$3x=-75-3$

$3x=-78$

$x=-26$

Q) $11x-4(2x-3)=24$

$11x-8x+12=24$

$3x=24-12$

$3x=12$

$x=4$

R) $3(4x-8)=3x+1$

$12x-24=3x+1$

$12x-3x=1+24$

$9x=25$

$x=\frac{25}{9}$

s) $2(x+3)=3(x-5)$

$2x+6=3x-15$

$2x-3x=-15-6$

$-x=-21$

$x=21$

t) $9x-8+12x=14-12x+3$

$9x+12x+12x=14+3+8$

$33x=25$

$x=\frac{25}{33}$

u) $1.4(x-5)=2.8(3x+5)$

$1.4x-7=8.4x+14$

$1.4x-8.4x=14+7$

$-7x=21$

$x=-3$

v) $(11x+7)-(7x-3)+(6x+1)=56$

$11x+7-7x+3+6x+1=56$

$10x=56+3-1$

$10x=45$

$x=\frac{45}{10}$

$x=\frac{9}{2}$

w) $\frac{1}{2}\left(\frac{1}{3}-\frac{1}{x}\right)=\frac{1}{4}$

$\frac{1}{6}-\frac{1}{2x}=\frac{1}{4}$

$-\frac{1}{2x}=\frac{1}{4}-\frac{1}{6}$

$\frac{1}{2x}=\frac{1}{12}$

$x=6$

x) $3x-(1-x)=5$

$3x-1+x=5$

$4x=5+1$

$4x=6$

$x=\frac{6}{4}$

$x=\frac{3}{2}$

y) $7x-(1-x)-(4-x)=5$

$7x-1+x-4+x=5$

$9x=5+4+1$

$9x=10$

$x=\frac{10}{9}$

z) $(11x+7)-(7x-3)+(6x+1)=56$

$11x+7-7x+3+6x+1=56$

$10x=56-7-3-1$

$10x=45$

$x=\frac{45}{10}$

$x=\frac{9}{2}$

zz) $12x-(6-9x)=9x+(-8)$

$12x-6+9x=9x-8$

$12x+9x-9x=-8+6$

$12x=-2$

$x=-\frac{2}{12}$

$x=-\frac{1}{6}$

2. Tim and his friend Sara together have $38. Tim has $4 more than Sara. Write an equation to represent how much money they have together. Indicate what your variable is.

$2x+4=38$ x is the amount of money that Sara has

$2x=38-4$

$2x=34$

$x=17$

Sara has $17

Tim has $21

3. Tom is 7 years older than his friend Jason. Together, they are 57 years old. Write an equation to represent how old they are together. Indicate what your variable is.

$2x+7=57$ x is Jason's age

$2x=57-7$

$2x=50$

$x=25$

Jason is 25 years old

Tom is 32 years old

4. There are 50 cupcakes in the staff room. Mr. CHEONG ate some of the cupcakes and Mr. Young ate twice as much as Mr. Cheong. Afterwards, there are 11 cupcakes left. Write an equation for how many cupcakes there are. What is your variable?

$50-3x=11$

x is the amount of cookies that Mr. Cheong ate

$-3x=11-50$

$-3x=-39$

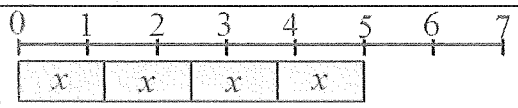
$x=13$

Mr Cheong ate 13 cookies

Mr Young ate 26 cookies

5. Given each diagram, find the length of each bar:

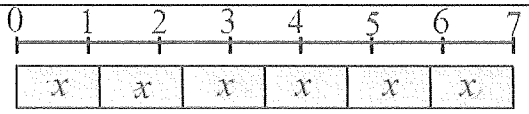
a)



$4x=5$

$x=\frac{5}{4}$

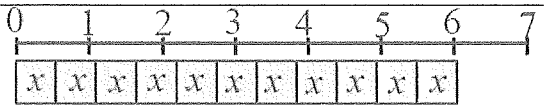
b)



$6x=7$

$x=\frac{7}{6}$

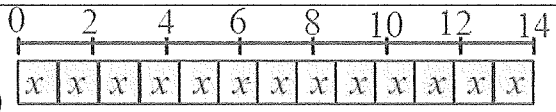
c)



$11x=6$

$x=\frac{6}{11}$

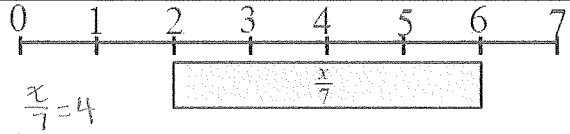
d)



$13x=14$

$x=\frac{14}{13}$

e)

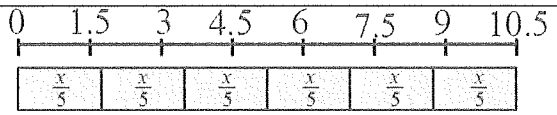


$\frac{x}{7}=4$

$\frac{x}{7}\times 7=4\times 7$

$x=28$

f)



$\frac{x}{5}=10.5$ $x=\frac{105}{12}$

$\frac{x}{5}\times\frac{21}{2}$

$12x=105$

6. Two positive integers are in the ratio of 8 to 13. If the difference between them is 35, find the larger integer.

$13x-8x=35$

$5x=35$

$x=7$

$7\times 8=56$

$7\times 13=91$

The larger integer is 91.

7. The lengths of fix of six line segments are $3x+1$, $2-2x$, $5x-1$, $4x-3$, *and* $3x+2$. Find the lengths of the sixth segment in terms of "x" if the mean of all six segments is $3x-2$ S is the length of the 6th segment

$3x+1+2-2x+5x-1+4x-3+3x+2=6(3x-2)-S$

$13x+1=18x-12-S$

$S=18x-12-13x-1$

$S=5x-13$

The length of the sixth segment is $5x-13$.

8. In the equation below, a student solves the equation by subtracting 5 from both sides. Is this step correct? Explain: $3x-5=12$

The student is wrong, the correct step is to add 5 on both sides to cancel out the negative five and isolate the $3x$.

$3x-5+5=12+5$

$3x=17$

$x=\frac{17}{3}$

9. Tim solves the equations below with the work shown. Explain if there are any errors:

S2 Tim didn't divide by 12.

$s1:\ 6x+12=24$

$s2:\ 6x+12=24$ (÷12 ÷12)

$s3:\ 6x+1=2$

$s4:\ 6x=1$

$s5:\ x=\frac{1}{6}$

S3: $\frac{1}{2}x+1=2$

S4: $\frac{1}{2}x=1$

S5: $x=2$

S2. Tim needed to subtract 1 from both sides, not add 1 to both sides.

$s1:\ \frac{5x}{3}+1=9$

$s2:\ \frac{5x}{3}=10$

$s3:\ 3\times\frac{5x}{3}=10\times3$

$s4:\ \frac{15x}{9}=30$

$s5:\ x=30\times\frac{9}{15}$

$s6:\ x=18$

S2: $\frac{5x}{3}=8$

S3: $3\times\frac{5x}{3}=8\times3$

S4. Tim multiplied the 3 with both the numerator and denominator instead of multiplying with only the numerator, or cancelling the denominator

$s1:\ 6x=7$

$s2:\ 6\times(6x)=7\times6$

$s3:\ 36x=42$

$s4:\ x=42-36$

$s5:\ x=6$

S4: $5x=24$

S5: $x=\frac{24}{5}$

S2. Not really a mistake, but it isn't necessary.

S4. $36x$ is $36(x)$, not $36+x$. It should be $\div$ instead of -36.

S4: $x=\frac{42}{36}$

S5: $x=\frac{7}{6}$

10. Given that $n!$ means the product of all natural numbers from "n" to 1, simplify the following:

$5!\left(\frac{1}{2!}-\frac{1}{3!}-\frac{1}{5!}\right)$

$=5\times4\times3\times2\left(\frac{1}{2}-\frac{1}{3\times2}-\frac{1}{5\times4\times3\times2}\right)$

$=120\left(\frac{1}{2}-\frac{1}{6}-\frac{1}{120}\right)$

$=120\left(\frac{60}{120}-\frac{20}{120}-\frac{1}{120}\right)$

$=120\left(\frac{39}{120}\right)$

$=39$

11. If $x+y=12$ and $x-y=8$, what is the value of $2x-xy$?

$x+y=12$

$+\ \ x-y=8$

$2x=20$

$x=10$

Substitute

$10+y=12$

$y=2$

$2\times10-10\times2=0$

(↑ x ↑ x ↑ y)

Name:_____ Date:_____

Math 8H HW Ch2 Lesson 3 Solving Equations with Fractions and Distributive Property

1. Distribute the term in front of the brackets to each term inside the brackets. Multiply and simplify:

a) $5(2x+4)$ $=10x+20$	b) $12(3x^2+8)$ $=36x^2+96$	c) $12(x-4)+3(4x-5)$ $=12x-48+12x-15$ $=24x-63$
d) $-3(2x^2-15x+11)$ $=-6x^2+45x-33$	e) $\frac{3}{4}(20x-12)$ $=15x-9$	f) $\frac{2}{3}(24x^2-18x+27)$ $=16x^2-12x+18$

2. Indicate LCD and then multiply all terms by the LCD. Simplify the equation and then solve:

a) $\frac{x}{5}+\frac{2}{3}=\frac{x}{3}$ LCD=15 $3x+10=5x$ $10=2x$ $5=x$	b) $\frac{x}{2}-\frac{1}{3}=\frac{2}{4}$ LCD=12 $6x-4=6$ $6x=10$ $x=\frac{5}{3}$
c) $\frac{2x}{3}=\frac{x}{2}+3$ LCD=6 $4x=3x+18$ $x=18$	d) $\frac{2x}{3}+\frac{4x}{5}-2=\frac{5x}{3}+3$ LCD=15 $10x+12x-30=25x+45$ $-3x=75$ $x=-25$

3. Solve the following equations for "x"

a) $x-\frac{1}{2}=-\frac{13}{2}$

$2x-1=-13$

$2x=-12$

$x=-6$

b) $-4\frac{1}{3}=x+\left(-\frac{1}{3}\right)$

$-\frac{13}{3}=x-\frac{1}{3}$

$-13=3x-1$

$-12=3x$

$x=-4$

c) $-(-4.25)+x=3$

$4.25+x=3$

$\boxed{x=-1.25}$

d) $\frac{1}{6}x-1=\frac{2}{3}$

$x-6=4$

$\boxed{x=10}$

e) $-\frac{7}{4}x+\frac{1}{2}=-\frac{9}{10}$

$-35x+10=-18$

$-35x=-28$

$x=\frac{35}{28}$

$\boxed{x=\frac{5}{4}}$

f) $-27=6\left(x+\frac{1}{2}\right)$

$-27=6x+3$

$-30=6x$

$\boxed{x=-5}$

g) $-18=\frac{-2}{5}(15x-45)$

$-18=-6x+18$

$6x=36$

$\boxed{x=6}$

h) $\frac{-5(2-x)}{3}=90$

$-5(2-x)=270$

$-10+5x=270$

$5x=280$

$\boxed{x=56}$

i) $5=\frac{3}{4}(3x+2)-2x$

$20=3(3x+2)-8x$

$20=9x+6-8x$

$\boxed{14=x}$

j) $10\left(\frac{1}{2}x-2\right)-\frac{2}{5}(30-5x)=-11$

$5x-20-12+2x=-11$

$7x=21$

$\boxed{x=3}$

i) $4(x+6)+2(5-x)-2[3-(a-2)]=4$

$4x+24+10-2x-2(3-a+2)=4$

$2x+34-6+2a-4=4$

$2x=2a+48$

$\boxed{x=a+48}$

k) $7x-6=x-5+4x$

$2x=1$

$\boxed{x=\frac{1}{2}}$

l) $20-4x+3=5x-8+2x$

$-11x=-33$

$\boxed{x=3}$

m) $3(x+6)+2x-6=8x-2(2x-4)$

$3x+18+2x-6=8x-4x+8$

$\boxed{x=-4}$

n) $18-(2x-5)-2(x+2)=3x+5$

$18-2x+5-2x-4=3x+5$

$-4x+19=3x+5$

$-7x=-14$

$\boxed{x=2}$

o) $\frac{1}{3}(3+9x)=\frac{7}{4}(28-12x)$

$1+3x=49-21x$

$24x=48$

$\boxed{x=2}$

p) $\frac{1}{6}(12x-42)=-\frac{3}{4}(4-16x)$

$2x-7=-3+12x$

$-10x=4$

$\boxed{x=-\frac{2}{5}}$

q) $24+\frac{1}{6}x=\frac{1}{3}x$

$-\frac{1}{6}x=-24$

$-x=-144$

$\boxed{x=144}$

r) $\frac{2(5x-1)}{3}=3x+1$

$2(5x-1)=9x+3$

$10x-2=9x+3$

$\boxed{x=5}$

s) $\frac{1}{2}x-6=\frac{3x}{10}+\frac{x}{8}$

$40x-240=12x+15x$

$3x=240$

$\boxed{x=80}$

t) $\frac{6(3-2x)}{5}=x+4$

$6(3-2x)=5x+20$

$18-12x=5x+20$

$-17x=2$

$\boxed{x=-\frac{2}{17}}$

u) $x+4=4\left[2(8x-7)+5x\right]-3x$

$x+4=4\left[16x-14+5x\right]-3x$

$x+4=64x-56+20x-3x$

$-80x=-60$

$\boxed{x=\frac{3}{4}}$

4. Challenge: Solve the following

a) $8+\frac{2}{5}(3x-1)=-2+\frac{1}{2}(20-3x)$

$80+4(3x-1)=-20+5(20-3x)$

$80+12x-4=-20+100-15x$

$27x=4$

$x=\frac{4}{27}$

b) $-0.4(0.5x-0.4)=1.3-0.6(0.3-0.2x)$

$-4(0.5x-0.4)=13-6(0.3-0.2x)$

$-2x+1.6=13-1.8+1.2x$

$-20x+16=130-18+12x$

$-32x=96$

$x=-3$

c) $2x-3(4x-6)=10-3[4(2x-5)+4x]$

$2x-12x+18=10-3[8x-20+4x]$

$-10x+18=10-24x+60-12x$

$26x=52$

$x=2$

d) $7x+2(3x-5)=6[5+2(2x-5+2x)]$

$7x+6x-10=6[5+4x-10+4x]$

$13x-10=30+24x-60+24x$

$-35x=-20$

$x=\frac{4}{7}$

e) $8+2[3x-2(x-8x+5)]=3(7x+3)-3x$

$8+2[3x-2x+16x-10]=21x+9-3x$

$8+6x-4x+32x-20=21x+9-3x$

$6x-4x+32x-21x+3x=9+20-8$

$16x=21$

$x=\frac{21}{16}$

f) $2-[3x+2(20x-4+24x+1)]=6(2x-3)-3x$

$2-[3x+40x-8+48x+2]=12x-18-3x$

$2-3x-40x+8-48x-2=12x-3x-18$

$-3x-40x-48x-12x+3x=-18+2-2-8$

$-100x=-26$

$x=\frac{26}{100}$

$x=\frac{13}{50}$

Name:_____ Date:_____

M8H HW CH2 Lesson 4: Using Equations to Model Problems

Write an equation for each of the questions below, indicate what your variables are and then solve:

1. Six times a number increased by 7 is 103. Find the number

$6x+7=103$ Let x be the number.

$6x=96$

$x=16$ The number is 16.

2. Two consecutive numbers have a sum of 47. Find the two numbers

$2x+1=47$ Let x be the smaller number

$2x=46$ The smaller number is 23.

$x=23$ The larger number is 24.

3. When 13 is subtracted from three-eighths of a number, the result is 11. Find the numbers.

$\frac{3}{8}x-13=11$ Let x be the number

$\frac{3}{8}x=24$ $x=64$

$x=24\times\frac{8}{3}$ The number is 64.

4. Bruce is 10 years older than Cindy. The sum of their ages is 52. How old is each person?

$2x+10=52$ Let x be Cindy's age

$2x=42$ Cindy is 21 years old.

$x=21$ Bruce is 31 years old.

5. For two consecutive numbers, the sum of the smaller and twice the larger is 38. Find the two numbers.

$x+2(x+1)=38$ Let x be the smaller number

$x+2x+2=38$ The smaller number is 12.

$3x=36$ The larger number is 13.

$x=12$

6. Three consecutive numbers have a sum of 159. Find the numbers.

$3x=159$ Let x be the middle number

$x=53$ The numbers are 52, 53, and 54.

$53-1=52$ $53+1=54$

7. Three consecutive even numbers have a sum of 672. Find the three numbers.

$3x=672$ Let x be the middle number

$x=224$ The numbers are 222, 224, and 226.

$224-2=222$ $224+2=226$

8. Mike ran twice as far as Brad. They ran a total of 18km. How far did each person run?

$3x=18$ Let x be the distance that Brad ran.

$x=6$ Brad ran 6 km.

Mike ran 12 km.

9. The difference of two numbers is 96. One number is nine time the other. What are the two numbers?

$9x-x=96$ Let x be the smaller number

$8x=96$ The numbers are 12 and 108.

$x=12$ * They can also be -12 and -108.

10. The sum of two numbers is 36. Four times the smaller is 1 less than the larger. What are the two numbers?

Let x be the smaller number $x+(36-x)=x$ $4x=36-x-1$ The two numbers are 7 and 29.

$5x=35$

$x=7$

11. Tom has equal number of nickels, dimes, and quarters. Their total value is $2.00. How many of each kind of coin does she have? Nickel = 5¢ Dime = 10¢ Quarters = 25¢

$(5+10+25)x = 200$ Let x be the amount of each kind of coin

$40x = 200$

$x = 5$ Tom has $\boxed{5}$ coins of each kind.

12. A collection of nickels and dimes has a total value of $8.50. How many coins are there if there are 3 times as many nickels as dimes? Let x be the amount of dimes

$5(3x)+10x = 850$ $34 \times 4 = 136$

$25x = 850$

$x = 34$ There are $\boxed{136}$ coins.

13. James has 91 coins which are nickels, dimes, and quarters. There are twice as many quarters as dimes, and half as many nickels as dimes. How much money does James have?

$2x+x+\frac{1}{2}x = 91$ Let x be the amount of dimes.

$\frac{7}{2}x = 91$ $10(26)+25(52)+5(13) = 1625$

$x = 26$ James has $\boxed{\$16.25}$.

14. Mike is four times as old as Ivy. The sum of their ages is 55 years. How old are they?

$x+4x = 55$ Let x be Ivy's age

$5x = 55$

$x = 11$ Ivy is $\boxed{11}$ years old.

Mike is $\boxed{44}$ years old.

15. Bob is twice as old as his brother Dave. In 7 years from now, Bob will be only one and one-half times as old as Dave. How old are they each now? Let x be Dave's current age

$2x+7 = \frac{3}{2}(x+7)$

$2x+7 = \frac{3}{2}x+\frac{21}{2}$ $x = 1$

$\frac{1}{2}x = \frac{7}{2}$

Dave is $\boxed{7}$ years old now.

Bob is $\boxed{14}$ years old now.

16. Robbie is three younger than Richard. Eight years ago, Robbie was one half of Richard's age. How old is each person now? Let x be Richard's current age

$x-3-8 = \frac{1}{2}(x-8)$

$x-11 = \frac{1}{2}x-4$ $x = 14$

$\frac{1}{2}x = 7$

Richard is $\boxed{14}$ years old now.

Robbie is $\boxed{11}$ years old now.

17. Sandy is four years less than twice Brad's age. In two years, Brad's age will be three-quarters of Sandy's age. How old is each person now? Let x be Brad's current age

$x+2 = \frac{3}{4}(2x-4+2)$

$x+2 = \frac{3}{2}x-3+\frac{3}{2}$ $x = 1$

$\frac{1}{2}x = \frac{7}{2}$

Brad is $\boxed{7}$ years old now.

Sandy is $\boxed{10}$ years old now.

18. The sum of three numbers is 33. The second number is 7 less than the first, and the third number is three times the second. What are the numbers?

$x+7+x+3x = 33$ Let x be the second number

$5x = 26$

$x = \frac{26}{5}$ The numbers are $\boxed{\frac{26}{5}/5.2,\ \frac{61}{5}/12.2,\ \text{and}\ \frac{183}{5}/36.6.}$

19. A BMW 325i travelled 1.2 times as fast as a Mercedes Benz. The difference in their speeds was 24km/h. Find the speeds of each car. Let x be the Mercedes Benz's speed

$1.2x-x=24$

$0.2x=24$

$2x=240$

$x=120$

The Mercedes Benz travelled at 120 km/h.

The BMW 325i travelled at 144 km/h.

20. The least o three consecutive integers is divided by 10, the next is is divided by 17, the greatest is divided by 26. What are the numbers if the sum of the quotients is 10? Let x be the smallest integer

$\frac{x}{10}+\frac{x+1}{17}+\frac{x+2}{26}=10$ $\quad x=50$

$442x+130x+130+85x+170=22100$

$457x=21800$

The numbers are 50, 51, and 52.

21. A stick is 60cm long and is cut into three pieces. The middle piece is 2cm longer than the shortest and 2cm shorter than the longest. How long is each piece?

$3x=60$ Let x be the length of the middle piece.

$x=20$

The shortest piece is 18 cm long.

The middle piece is 20 cm long.

The longest piece is 22 cm long.

22. Explain why every integer can be expressed in exactly one of the four forms: $6n,\ 6n+1,\ 6n+2,\ 6n+3,\ 6n+4,\ 6n+5$

b) In which of these 5 forms can the prime numbers be expressed? Explain.

c) Show that all prime numbers (except 2 and 3)when divided by 6 leave a remainder of either 1 or 5.

c) When a prime number is divided by 6, the remainder will always be odd, but it cannot be 3 because then it will be a multiple of 3

Let n be the quotient when an integer is divided by 6

a) It is because when a integer is divided by 6, it will either have a remainder of 0 $(6n)$, 1 $(6n+1)$, 2 $(6n+2)$, 3 $(6n+3)$, 4 $(6n+4)$, or 5 $(6n+5)$.

b) $6n+1$ & $6n+5$ because when a prime number is divided by 6, it will always have an odd remainder, while the form $6n+3$ represent multiples of 3.

23. Ray has a box of candy bars. He gave Mike half of what he had plus half a bar. Then he gave Chris half of what he had left plus half a bar. After which he gave Larry half of what he had left plus a bar. Then finally again, he gave Andy half of what he had left plus half a bar. Then he had no bars left. How many candy bars did Ray have in the beginning?

Let x be the amount of candy bars Ray had in the beginning.

Beginning	Gave	Left
x	$\frac{x}{2}+\frac{1}{2}$ (M)	$\frac{x}{2}-\frac{1}{2}$
$\frac{x}{2}-\frac{1}{2}$	$\frac{x}{4}+\frac{1}{4}$ (C)	$\frac{x}{4}-\frac{3}{4}$
$\frac{x}{4}-\frac{3}{4}$	$\frac{x}{8}+\frac{3}{8}$ (L)	$\frac{x}{8}-\frac{11}{8}$
$\frac{x}{8}-\frac{11}{8}$	$\frac{x}{16}-\frac{3}{16}$ (A)	0

$\frac{x}{8}-\frac{11}{8}=\frac{x}{16}-\frac{3}{16}$ $\quad x=19$

$\frac{2x}{16}-\frac{22}{16}=\frac{x}{16}-\frac{3}{16}$

$\frac{x}{16}=\frac{19}{16}$

Ray had 19 candy bars in the beginning.

24. Suppose "m" and "n" are positive odd integers. Which of the following must also be an odd integer?

(A) $m+3n$ (B) $3m-n$ (C) $3m^2+3n^2$ (D) $(nm+3)^2$ (E) $3mn$

It is E because the product of 2 odd integers is always odd.

25. Let "a', "b", and "c" be numbers with 0 < a < b < c. Which of the following is impossible?

(A) $a - c < b$ (B) $a \cdot b < c$ (C) $a + b < c$ (D) $a \cdot c < b$ (E) $\frac{b}{c} = a$

26. What is the least positive integer "n" such that "n" is a multiple of 6 and neither *"n - 1" or "n + 1"* is a prime number?

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120

It is 120 because $119 = 7 \times 17$ and $121 = 11 \times 11$.

27. Assume that weights of coins as follows:

1 Cent coin (penny) – 3grams; 5 cent coin (nickel) 5 grams; 10 cent coin (dime) 2 grams;
25 cent coin (quarter) - 9 grams; 1 dollar coin (loonie) - 13 grams; 2 dollar coin (toonie) – 17 grams.

Dan holds as least one of each of the above coins, which a total weight of exactly 220 grams. What is the maximum possible total value of his coins? Give your answer in dollars to the nearest cent.

← Highest ¢/g

Penny = $\frac{1}{3}$¢/g Nickel = 1¢/g Dime = 5¢/g Quarter = $\frac{25}{9}$¢/g Loonie = $\frac{100}{13}$¢/g Toonie = $\frac{200}{17}$¢/g

1+5+10+25+100+200 = 346¢/49g 171g left 8t = 1600¢/153g → 1946¢/202g 18g left

+ HL+1D+1P = 2057¢/220g

The maximum possible total value of his coins is 2057 cents.

28. In the multiplication problem below, A, B, C, and D are different digits. What is the value of *A + B*?

$$\begin{array}{r} ABA \\ \times\ \ CD \\ \hline CDCD \end{array}$$

(A) 1 (B) 2 (C) 3 (D) 1 (E) 9

ABA = 101 $101 \times xy = xyxy$

29. In the multiplication problem below, the letters G, M, A, T, and H represent different digits. What is the value of G + M + A + T + H?

$$\begin{array}{r} 2\ 0\ 0\ 8 \\ \times\ \ \ \ H\ T \\ \hline G\ M\ A\ T\ H \end{array}$$

① H = 1, 2, 3, 4, → 2008 × 50 = 100400

② 8 × T = ?4 H = 2, 4 → 24, 32, 64, 72 → T = 3, 4, 8, 9

③ Start guessing, since there are only a few options

④ T = 8, H = 4 $\left(\begin{array}{r} 8 \\ \times\ 48 \\ \hline 3864 \end{array}\right)$ 2008 × 48 = 96384 9+6+3+8+4 = 30

The value of G+M+A+T+H is 30.

30. Mr. Young divided $45 among four students: Andy, Bob, Chris, and Dianna. When the students complained that the shares were not equal, he instructed Bob to give Andy $2. Then he doubled Chris' share and cut Dianna's share in half. Now all the students have the same amount. How much money do they have in total?

Beginning	A	B	C	D
End	A+2	B-2	2c	$\frac{D}{2}$

$A+2=B-2=2c=\frac{D}{2}$

Change variables to A: $B\to A$ $A+2=B-2$, $A+4=B$; $C\to A$ $A+2=2c$, $\frac{1}{2}A+1=C$; $D\to A$ $A+2=\frac{D}{2}$, $2A+4=D$

$A+A+4+2A+4+\frac{1}{2}A+1=45$ Let A be Andy's money in the beginning

$\frac{9}{2}A+9=45$

$\frac{9}{2}A=36$

$A=8$

$(8+2)4=40$

They have $40 in total.

31. Suppose that

$ab=6$, $bc=8$, $cd=10$, and $de=12$.

Wat is the value of $\frac{a}{e}$? Express your answer as a common fraction

$a\cdot b=6 \longrightarrow b=\frac{6}{a}$

$\frac{6}{a}\cdot c=8 \longrightarrow c=\frac{4a}{3}$

$\frac{4a}{3}\cdot d=10 \longrightarrow d=\frac{15}{2a}$

$\frac{15}{2a}\cdot e=12 \longrightarrow e=\frac{8a}{5}$

$\frac{b}{a}\cdot\frac{4a}{3}=2$

$8a=8$

$a=8$

$\frac{8}{\frac{8(8)}{5}}=\frac{8}{\frac{64}{5}}=8\div\frac{64}{5}=8\times\frac{5}{64}=\frac{5}{8}$

The value of $\frac{a}{e}$ is $\frac{5}{8}$.

32. Challenge:

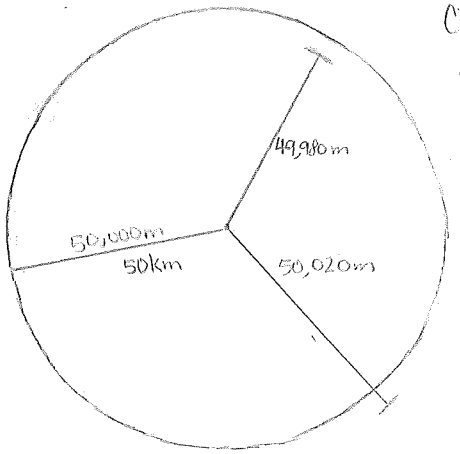
King Charles the Short-Sighted rode all the way around the boundary of his kingdom, which is a circle with diameter 100 kilometres. While he rode along, he could see only 20 metres in any direction. What is the total area (both inside and outside his kingdom) of the region that he saw? Assume the world is flat. Express your answer in terms of π.

$50020^2\pi-49980^2\pi$

$=2502000400\pi-2498000400\pi$

$=4000000\pi$

King Charles the Short-Sighted saw 4000000π m^2 of region.



Circle line → Kingdom boundary

Gray → Area he saw

Name: Francis Date:

M8H HW Ch2 Lesson 5: Solving Equations Involving Reciprocals and Square Roots

1. Solve the following equations and check your answers:

a) $x=\frac{1}{x}$

$x^2=1$

$x=1$

b) $\frac{3}{b}=\frac{1}{12}$

$b=36$

c) $\frac{a}{5}=\frac{5}{a}$

$a^2=25$

$a=5$

d) $\frac{2}{y}=\frac{3}{\left(\frac{1}{2}\right)}$

$1=3y$

$y=\frac{1}{3}$

e) $4=\frac{2}{3+\frac{1}{x}}$

$12+\frac{4}{x}=2$

$\frac{4}{x}=-10$

$\frac{4}{10}=x$

$-\frac{2}{5}=x$

f) $\frac{1\frac{1}{4}}{x}=\frac{8}{3}$

$\frac{\frac{5}{4}}{x}=\frac{8}{3}$

$8x=\frac{15}{4}$

$x=\frac{15}{32}$

g) $\frac{8}{x}=\frac{x}{18}$

$x^2=144$

$x=12$

h) $\frac{4}{x}=\frac{\left(\frac{1}{9}\right)}{\left(\frac{1}{x}\right)}$

$\frac{4}{x}=\frac{x}{9}$

$x^2=36$

$x=6$

i) $\frac{3\frac{1}{2}}{4\frac{2}{3}}=\frac{9\frac{3}{7}}{n}$

$\frac{\frac{7}{2}}{\frac{14}{3}}=\frac{66}{n}$

$\frac{7}{2}n=308$

$7n=616$

$n=88$

j) $\frac{2}{y+5}=\frac{7}{y}$

$2y=7y+35$

$-5y=35$

$y=-7$

k) $\frac{2}{n+10}=\frac{6}{n}$

$6n+60=2n$

$4n=-60$

$n=-15$

l) $\frac{1-x}{4+x}=\frac{2}{3}$

$8+2x=3-3x$

$5x=-5$

$x=-1$

m) $\frac{2}{y+5}=\frac{7}{y}$

$7y+35=2y$

$5y=-35$

$y=-7$

n) $\frac{2+\frac{12}{x}}{\frac{2}{x}+3}=4$

$\frac{8}{x}+12=2+\frac{12}{x}$

$-\frac{4}{x}=-10$

$\frac{4}{10}=x$

$x=\frac{2}{5}$

o) $\frac{4+\frac{3}{x}}{\frac{3}{x}+\frac{1}{3}}=3$

$\frac{9}{x}+1=4+\frac{3}{x}$

$\frac{6}{x}=3$

$\frac{6}{3}=x$

$x=2$

p) $\frac{\frac{1}{4}-\frac{2}{x}}{\frac{2}{x}-\frac{1}{4}}=y$

$\frac{\frac{1}{4}-\frac{2}{x}}{\frac{2}{x}-\frac{1}{4}}=-1$

$y=-1$

2. Solve the following equations by eliminating the root function. Remember to check your solutions!!

a) $3=\sqrt{2+x}$ $9=2+x$ $x=7$	b) $5=2+\sqrt{4-x}$ $3=\sqrt{4-x}$ $9=4-x$ $x=-5$	c) $\sqrt{2x-1}=4$ $2x-1=16$ $2x=17$ $x=\frac{17}{2}$	d) $3-\sqrt{\frac{x}{2}+3}=1$ $\sqrt{\frac{x}{2}+3}=2$ $\frac{x}{2}+3=4$ $\frac{x}{2}=1$ $x=2$
e) $3+\sqrt{2x+3}=2$ $\sqrt{2x+3}=-1$ A square root cannot be negative, therefore there is No Solution	f) $\frac{1}{\sqrt{2x+1}}-2=\frac{3}{2}$ $\frac{1}{\sqrt{2x+1}}=\frac{7}{2}$ $x=-\frac{45}{98}$ $\sqrt{2x+1}=\frac{2}{7}$ $2x+1=\frac{4}{49}$ $2x=-\frac{45}{49}$	g) $\sqrt{2+\sqrt{x}}=2$ $2+\sqrt{x}=4$ $\sqrt{x}=2$ $x=4$	h) $\sqrt{1+\sqrt{2+\sqrt{x}}}=3$ $1+\sqrt{2+\sqrt{x}}=9$ $2+\sqrt{x}=8^2$ $\sqrt{x}=62$ $x=62^2$ $x=3844$
i) $\sqrt{\sqrt{\sqrt{3x}}}=4$ $\sqrt{\sqrt{3x}}=16$ $\sqrt{3x}=64$ $3x=256$ $x=\frac{256}{3}$	j) $\frac{2}{\sqrt{x}}=\frac{4}{x}$ $2x=4\sqrt{x}$ $4x^2=16x$ $x^2=4x$ $x=4$	k) $\sqrt{2-2x+1}=4$ $\sqrt{2-2x}=3$ $2-2x=9$ $-2x=7$ $x=-\frac{7}{2}$	l) $2-\sqrt{1-4x}=1$ $-\sqrt{1-4x}=-1$ $\sqrt{1-4x}=1$ $1-4x=1$ $-4x=0$ $x=0$

3. Isolate the indicated variable in each expression:

a) $\frac{A}{B}=\frac{C}{D}$; for C $\frac{A}{B}\cdot D=C$ $C=\frac{AD}{B}$		c) $\frac{A}{B}=\frac{C}{D}$; for B $B=\frac{C}{D}\cdot\frac{1}{A}$ $B=\frac{C}{AD}$	d) $\frac{A}{B}+\frac{A}{C}=1$; for A $AC+AB=BC$ $A(C+B)=BC$ $A=\frac{BC}{B+C}$
e) $\frac{A}{B}+\frac{A}{C}=\frac{1}{BC}$; for A $AC+AB=1$ $A(C+B)=1$ $A=\frac{1}{C+B}$	f) $\frac{A}{B}+\frac{A}{F}=\frac{C}{D}$; for A $AFD+ABD=CBF$ $A(FD+BD)=CBF$ $A=\frac{CBF}{FD+BD}$	g) $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}=\frac{1}{D}$; for D $\frac{BC+AC+AB}{ABC}=\frac{1}{D}$ $\frac{ABC}{BC+AC+AB}=D$	h) $\frac{\frac{1}{A}+\frac{1}{B}}{\frac{1}{A}-\frac{1}{B}}=C$; for A $\frac{B+A}{B-A}=C$ $A=\frac{CB-B}{C+1}$ $B+A=BC-AC$ $A+AC=BC-B$ $A(1+C)=BC-B$

4. Solve the following equations:

a) $\dfrac{3}{4+\dfrac{x}{3+\dfrac{1}{7}}}=1$

$4+\dfrac{x}{\frac{22}{7}}=3$

$\dfrac{7}{22}x=-1$

$7x=-22$

$\boxed{x=-\frac{22}{7}}$

b) $1+\dfrac{x}{2+\dfrac{1}{1-\dfrac{1}{2}}}=7$

$\dfrac{x}{2+\frac{1}{1-\frac{1}{2}}}=6$

$\dfrac{x}{2+2}=6$

$\frac{x}{4}=6$

$\boxed{x=24}$

c) $5\frac{4}{5}=1+\dfrac{x}{1+\dfrac{1}{1+\dfrac{1}{1+1}}}$

$4\frac{4}{5}=\frac{x}{1+\frac{1}{1+\frac{1}{2}}}$

$4\frac{4}{5}=\frac{x}{1+\frac{1}{\frac{3}{2}}}$

$4\frac{4}{5}=\frac{x}{1+\frac{2}{3}}$

$\frac{24}{5}=\frac{x}{\frac{5}{3}}$

$5x=40$

$\boxed{x=8}$

d) $\dfrac{1}{1+\dfrac{2}{1+\dfrac{3}{1+x}}}=2$

$1+\frac{2}{1+\frac{3}{1+x}}=\frac{1}{2}$

$\frac{2}{1+\frac{3}{1+x}}=-\frac{1}{2}$

$-4=1+\frac{3}{1+x}$

$-5=\frac{3}{1+x}$

$1+x=-\frac{3}{5}$

$\boxed{x=-\frac{8}{5}}$

e) $1+\dfrac{2+\dfrac{12}{x}}{\dfrac{x}{2}+3}=\dfrac{2}{x}$

$2\left\{\frac{2+\frac{12}{x}}{\frac{x}{2}+3}\right.=\frac{2}{x}$

$\frac{4+\frac{24}{x}}{x+6}=\frac{2}{x}$

$4x+24=2x+12$

$x^2+8x=-12$

$x^2+8x+12=0$

$-8\pm\sqrt{64-48}$

$=\frac{-8\pm\sqrt{16}}{2}$

$=\frac{-8\pm4}{2}$

$x=-6$ $\boxed{x=-2}$

f) $\dfrac{1}{\sqrt{1+\dfrac{1}{\sqrt{1+\sqrt{x}}}}}=\dfrac{1}{3}$

$\sqrt{1+\frac{1}{\sqrt{1+\sqrt{x}}}}=3$

$1+\frac{1}{\sqrt{1+\sqrt{x}}}=9$

$\frac{1}{\sqrt{1+\sqrt{x}}}=8$

$\sqrt{1+\sqrt{x}}=\frac{1}{8}$

$\frac{1}{1+\sqrt{x}}=\frac{1}{64}$

$1+\sqrt{x}=64$

$\sqrt{x}=63$

$x=63^2$

$\boxed{x=3969}$

5. Suppose that "a", "b", "c", and "d" are positive integers such that the equation below is true, what is the value of $a + b + c + d$?

$$a+\dfrac{1}{b+\dfrac{1}{c+\dfrac{1}{d}}}=\dfrac{37}{11}$$

$a+\frac{1}{b+\frac{1}{c+\frac{1}{d}}}=3\frac{4}{11}$ ($a=3$)

$\frac{1}{b+\frac{1}{c+\frac{1}{d}}}=\frac{4}{11}$

$b+\frac{1}{c+\frac{1}{d}}=\frac{11}{4}=2\frac{3}{4}$ ($b=2$)

$\frac{1}{c+\frac{1}{d}}=\frac{3}{4}$

$c+\frac{1}{d}=\frac{4}{3}=1\frac{1}{3}$ ($c=1$)

$\frac{1}{d}=\frac{1}{3}$

$d=3$

$a=3$ $b=2$ $c=1$ $d=3$

The value of $a+b+c+d$ is $\boxed{9}$.

6. Solve for "x"

$$4=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots..}}}}$$

$16 = x+\sqrt{x+\sqrt{x}}\ \ldots$

$16 = x+4$

$\boxed{x=12}$

7. Solve for "x"

$$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\frac{1}{5\cdot6}+\ldots.+\left(\frac{1}{2008\cdot2009}\right)+\left(\frac{1}{2009\cdot2010}\right)=\frac{1}{x}$$

$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)\ldots+\left(\frac{1}{2008}-\frac{1}{2009}\right)+\left(\frac{1}{2009}-\frac{1}{2010}\right)=\frac{1}{x}$

Cancelled $\pm\frac{1}{2}$ to $\pm\frac{1}{2009}$ $\rightarrow$ $\left(\frac{1}{1}-\frac{1}{2010}\right)=\frac{1}{x}$

$\frac{2009}{2010}=\frac{1}{x}$

$\frac{2010}{2009}=\frac{x}{1}$

$\boxed{x=\frac{2010}{2009}}$

8. Challenge: Solve for "x"

$\sqrt{x-1}+4=3+\sqrt{x+1}$

$\sqrt{x-1}+1=\sqrt{x+1}$ Square both sides

$(\sqrt{x-1}+1)(\sqrt{x-1}+1)=x+1$

$(x-1)+2\sqrt{x-1}+1=x+1$

$2\sqrt{x-1}=1$ square again

$4(x-1)=1$

$4x-4=1$

$4x=5$

$x=\frac{5}{4}$

$\boxed{x=\frac{5}{4}}$

$\sqrt{\left(\frac{5}{4}\right)-1}+4=3+\sqrt{\left(\frac{5}{4}\right)+1}$

$\sqrt{\frac{1}{4}}+4=3+\sqrt{\frac{9}{4}}$

$\frac{1}{2}+4=3+\frac{3}{2}$

$\frac{9}{2}=\frac{9}{2}$

Name:_____ Date:_____

M8H HW Ch2 Lesson 6: Simplifying Polynomials - Distributions and FOIL

1. Indicate whether if the following pairs are like terms or not. If they are, simplify the expression:

a) $3ab+5ba$ Yes $=8ab$	b) $5a^2b+8ab^2$ No	c) $9^2ab+3ab$ Yes $=81ab+3ab$ $=84ab$	d) $12abc-11cab$ Yes $=abc$
e) $4xyz+9xy^2$ No	f) $8(a+b)-2ab$ No	g) $\frac{1}{2ab}+3ab$ No	h) $\frac{1}{3}cd+3d^2c$ No
i) $3mn+5(nm+1)$ Yes $=3mn+5nm+5$ $=8nm+5$	j) $3a^3b^2-12b^3a^2$ No	k) $12y^3x^2z^2v-5z^2x^2y^3w$ No $V \neq W$	l) $\frac{3a^3b}{a}+8ba^2$ Yes $=3a^3b+8ba^3$ $=11a^3b$

2. Expand and Simplify each of the following equations:

a) $3(2x+3y+1)$ $=6x+9y+3$	b) $3a(2a^2+3ab)$ $=6a^3+9a^2b$	c) $4a-3(2a+1)$ $=4a-6a-3$ $=-2a-3$
d) $6(3-2x)+7x$ $=18-12x+7x$ $=18-5x$	e) $2(2x+1)-3(1-2x)$ $=4x+2-3+6x$ $=10x-1$	f) $3(2+4x)-7x$ $=6+12x-7x$ $=6+5x$
g) $-4(2x+x-x)-3x$ $=-8x-4x+4x-3x$ $=-11x$	h) $3(x-1)+2(2x+1)$ $=3x-3+4x+2$ $=7x-1$	i) $-3(-5x-1)+2x(1-x)$ $=15x+3+2x-2x^2$ $=17x+3-2x^2$
j) $3x(6x+3y+1)-4y(2x)$ $=18x^2+9xy+3x-8xy$ $=18x^2+xy+3x$	k) $-3x(4x+3y)-9y(2x+1)$ $=-12x^2-9xy-18xy-9y$ $=-12x^2-27xy-9y$	l) $7x^2(2y-1)-8y(3x^2-x)$ $=14x^2y-7x^2-24x^2y+8xy$ $=-10x^2y-7x^2+8xy$

m) $3(2ab-2b-a)-7ab-7b$ $=6ab-6b-3a-7ab-7b$ $=-ab-13b-3a$	n) $-3ab(7a+12b+11)-7b(a^2+10ab-2)$ $=-21a^2b-36ab^2-33ab-7a^2b-70ab^2+14b$ $=-28a^2b-106ab^2-33ab+14b$
o) $3(x-y)+8(y-2x)-3(3y+3)$ $=3x-3y+8y-16x-9y-9$ $=-13x-4y-9$	p) $2x(3x^2+1)-3x^2(3-3x)-8(3x^2+9x^3)$ $=6x^3+2x-9x^2+9x^3-24x^2-72x^3$ $=57x^3-33x^2+2x$
q) $-11(3x-8y)-5(3y-9x)-4(11y-12+3x)$ $=-33x+88y-15y+45x-44y+48-12x$ $=29y+48$	r) $-11(3x-8y)-5(8y-3x)-4(13x-8y)$ $=-33x+88y-40y+15x-52x+32y$ $=-70x+80y$

3. FOIL and Simplify the following

a) $2x(2x+3)$ $=4x^2+6x$	b) $4x(5x-3)$ $=20x^2-12x$	c) $(2x+1)(4x)$ $=8x^2+4x$
d) $(2x-3)(4x+1)$ $=8x^2+2x-12x-3$ $=8x^2-10x-3$	e) $(6x-3)(3x+2)$ $=18x^2+6x-9x-6$ $=18x^2-3x-6$	f) $(4x-3)(4x+3)$ $=16x^2+12x-12x-9$ $=16x^2-9$

g) $(3x+4)(2x-3)$ $=6x^2-9x+8x-12$ $=6x^2-x-12$	h) $(4x-1)(3x+2)$ $=12x^2+8x-3x-2$ $=12x^2+5x-2$	i) $(7x-1)(4x-5)$ $=28x^2-35x-4x+5$ $=28x^2-39x+5$
j) $(12x-3)(8x-10)$ $=96x^2-120x-24x+30$ $=96x^2-144x+30$	k) $(6x-8)(4x+6)$ $=24x^2+36x-32x-48$ $=24x^2+4x-48$	L) $(2x+1)(3x^2+5x-6)$ $=6x^3+10x^2-12x+3x^2+5x-6$ $=6x^3+13x^2-7x-6$

4. Given that the area of a rectangle is represented by: $x^2+7x+12$, find an expression for the length and width in terms of "x".

$x^2+7x+12=(x+3)(x+7)$

The length and width of the rectangle is $x+3$ and $x+4$.

5. A large rectangle has a length of 3x+8y and a width of 5x+2y. A small square with lengths of 2x+y is cut out of the rectangle. What is the area of the remaining shape?

$(3x+8y)(5x+2y)-(2x+y)(2x+y)$

$=(15x^2+6xy+40xy+16y^2)-(4x^2+2xy+2xy+y^2)$

$=15x^2+46xy+16y^2-4x^2-4xy-y^2$

$=11x^2+42xy+15y^2$

The area of the remaining shape is $\boxed{11x^2+42xy+15y^2}$.

6. Given that the width of a rectangle is: $6x$ and the area is: $6x^3+30x^2+24x$, what is the length of the rectangle in terms of "x"?

$$\frac{6x^3+30x^2+24x}{6x}$$

$=x^2+5x+4$

The length of the rectangle is $\boxed{x^2+5x+4}$.

Getting $6x^3$ (highest power of x in this equation → x^3) using x^2 and 3 (lowest power of x in this equation → x^0) using 1 (x^0)

Finding the remaining part of the length. Already found x^3 and x^0, so now we need to find x^1

7. Challenge: Given that the width of a rectangle is: $(6x+3)$ and the area is: $6x^3+33x^2+21x+3$, what is the length of the rectangle in terms of "x"?

① $(6x+3)(x^2+1)=6x^3+3x^2+6x+3$

② $6x^3+33x^2+21x+3=6x^3+3x^2+6x+3+(30x^2+15x)$

③ $\frac{30x^2+15x}{6x+3}=\frac{5x(6x+3)}{6x+3}=5x$

④ $(6x+3)(x^2+5x+1)=6x^3+30x^2+6x+3x^2+15x+3=6x^3+33x^2+21x+3$

The length of the rectangle is x^2+5x+1

8. Given the area of each rectangle, find the area of the unknown rectangle and the dimensions of each side:

	x	6
x	x^2	$6x$
-4	$-4x$	?

? $=-24$

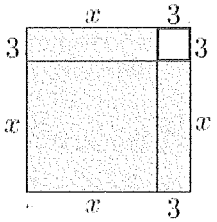
	$2x$	3
x	$2x^2$	?
-2	$-4x$	-6

? $=3x$

	$-3x$	2
$-2x$	$6x^2$	$-4x$
5	$-15x$	?

? $=10$

9. If the shaded region below has area $150cm^2$, what is the value of "x"?



$x^2+6x=150$

$x^2+6x-150=0$

$x=\frac{-6\pm\sqrt{36+600}}{2}$

$x=\frac{-6\pm\sqrt{636}}{2}$

$x=\frac{-6\pm\sqrt{-25.219}}{2}$

$x=\sim\frac{19.219}{2}$ ← x can only be positive, there is no negative distance

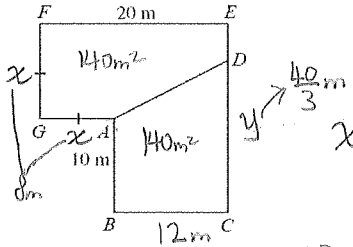
$x=\sim 9.609$

13.

In the diagram, $ABCDEFG$ is a room having square corners, with $EF=20$ m, $AB=10$ m, and $AG=GF$. The total area of the room is 280 m^2. A wall is built from A to D creating two rooms of equal area. What is the distance, in metres, from C to D?

(A) 15 (B) $\frac{50}{3}$ (C) 12

(D) 13 (E) $\frac{40}{3}$



	x	$20-x$
x	x^2	$20x-x^2$
10	$10x$	$200-10x$

$x^2+20x-x^2+200-10x=280$

$10x+200=280$

$10x=80$

$x=8$

$\frac{(10+y)\cdot 12}{2}=140$

$(10+y)\cdot 6=140$

$60+6y=140$

$6y=80$

$3y=40$

$y=\frac{40}{3}$

Name:_____ Date:_____

M8H HW CH2 Lesson 7 Factoring GCF and Trinomials

1. Find the GCF and then factor out the GCF

a) $8x^2y-26xy^2$ GCF=2xy $=2xy(4x-13y)$	b) $25x^3y^2+55xy^2+10xy$ GCF=5xy $=5xy(5x^2y+11y+2)$
c) $10xyz^2+15x^3z^5-35x^2yz^2$ GCF=$5xz^2$ $=5xz^2(2y+3x^2z^3-7xy)$	d) $24bc^3-72a^3bc^2+88a^2b^3c$ GCF=8bc $=8bc(3c^2-9a^3c+11a^2b^2)$
e) $x^2(2x-1)+5x(1-2x)$ $=2x^3-x^2+5x-10x^2$ $=x(2x^2-x+5-10x)$	f) $8x^2(6x-10)+12x^3(3x-5)$ $=48x^3-80x^2+24x^4-60x^3$ $=4x(12x^2-20x+6x^3-15x^2)$
g) $27x^3y^2+18x^2y^3-45xy^4$ GCF=$9xy^2$ $=9xy^2(3x^2y+2xy-5y^2)$	h) $18xy^2(2-3x)-12xy(15-10x)+9x^2y(6-4x)$ $=36xy^2-54x^2y^2-180xy+120x^2y+54x^2y-36x^3y$ $=6xy(6y-9xy-30+20x+9x-6x^2)$ $=6xy(6y-9xy-30+29x-6x^2)$

2. Given each expression, find the missing value in the box:

a. $x^2-11x-12=(x-\boxed{?})(x+1)$

$x^2-11x-12=(x-12)(x+1)$

$?=12$

b. $x^2-29x+120=(x-\boxed{?})(x-5)$

$x^2-29x+120=(x-24)(x-5)$

$?=24$

c. $x^2-13x+12=(x+\boxed{?})(x+\boxed{?})$

$x^2-13x+12=(x-12)(x-1)$

$?=-12,-1$

d. $x^2-\overset{y}{\boxed{?}}x+44=(x-\overset{z}{\boxed{?}})(x-11)$

$(-11)(-4)=44$ $-11-4=-15$

$z=4$ $y=15$

e. $x^2-2x-15=(x+\boxed{?})(x+\boxed{?})$

$x^2-2x-15=(x-5)(x+3)$

$?=-5,3$

f. $x^2-12x+\overset{y}{\boxed{?}}=(x-\overset{z}{\boxed{?}})(x-7)$

$-7-5=-12$ $(-7)(-5)=35$

$z=5$ $y=35$

g) $x^2-15x-\overset{y}{\boxed{?}}=(x+5)(x+\overset{z}{\boxed{?}})$

$5-20=-15$ $(-20)(5)=-100$

$z=-20$ $y=100$

h) $3x^2-3x-36=(3x+\overset{y}{\boxed{?}})(x+\overset{z}{\boxed{?}})$

$3(x^2-x-12)=3(x+\frac{y}{3})(x+z)$

$x^2-x-12=(x+3)(x-4)$

$3x^2-3x-36=(3x+9)(x-4)$

$y=9$ $z=-4$

3. Factor each of the following expressions. Show all your steps and work:

a. x^2+7x+6 $=(x+6)(x+1)$	b. $x^2+25x+24$ $=(x+24)(x+1)$
c. $x^2+10x+21$ $=(x+3)(x+7)$	d. $x^2-10x+24$ $=(x-6)(x-4)$
e. $x^2+3x-40$ $=(x+8)(x-5)$	f) $3x^2-15x-42$ $=3(x-5x-14)$ $=3(x-7)(x+2)$
g) $4x^2-36x+72$ $=4(x^2-9x+18)$ $=4(x-6)(x-3)$	h) x^3+8x^2-20x $=x(x^2+8x-20)$ $=x(x+10)(x-2)$
i) $2x^2y+28xy+42y$ $=2y(x^2+14x+21)$	j) $3x^3y-81x^2y+150xy$ $=3xy(x^2-27x+50)$ $=3xy(x-25)(x-2)$
k) $5x^2+20xy-105y^2$ $=5(x^2+4xy-21y^2)$ $=5(x+7y)(x-3y)$	L) x^4-8x^2+12 $=(x^2-2)(x^2-6)$
m) $-x^4-10x^2-21$ $=-(x^4+10x^2+21)$ $=-(x^2+3)(x^2+7)$	n) $-x^4-4x^2y+45y^2$ $=-(x^4+4x^2y-45y^2)$ $=-(x^2+9y)(x^2-5y)$

o) $-2x^4+8x^2y^2-154y^4$

$=-2(x^4-4x^2y^2+77y^4)$

$=-2(x^2-7y^2)(x^2+4y^2)$

p) $3x^4-3x^2y^2-216y^4$

$=3(x^4-x^2y^2-72y^4)$

$=3(x^2-9y^2)(x^2+8y^2)$

q) $2x^2(3-2x)+24x(2x-3)+54(3-2x)$

$=2x^2(3-2x)-24x(3-2x)+54(3-2x)$

$=(3-2x)(2x^2-24x+54)$

$=2(3-2x)(x^2-12x+27)$

$=2(3-2x\quad -3)(x-9)$

r) $2x^2(x-y)-14xy(x-y)+88y^2(y-x)$

$=2x^2(x-y)-14xy(x-y)-88y^2(x-y)$

$=(x-y)(2x^2-14xy-88y^2)$

$=2(x-y)(x^2-7xy-44y^2)$

$=2(x-y)(x-11y)(x+4y)$

4. Evaluate the following by factoring and without using a calculator

a) $\dfrac{17^2-17+6}{17+2}$ ← only works when this is a minus sign

$\dfrac{17^2-17-6}{17+2}$

$=\dfrac{(17-3)(17+2)}{17+2}$

$=17-3 \quad =14$

b) $(15)^2+17(15)+60$

$=[(15)+5][(15)+12]$

$=20\cdot 27$

$=540$

c) $\dfrac{17^2-9(17)+20}{(17)+5}$ ← This should be −5

$=\dfrac{[(17)-4][(17)-5]}{(17)-5}$

$=(17-4)=13$

d) $\dfrac{17^3+2(17)^2+34(17)}{17^2}$

$=\dfrac{17^3+2(17)^2+2(17)(17)}{17^2}$

$=17+2+2$

$=21$

e) $\dfrac{(13)^2-4(13)(11)-12(11)}{(13)+2(11)}$

$=\dfrac{[(13)-6(11)][(13)+2(11)]}{(13)+2(11)}$

$=[13-6(11)]$

$=-53$

f) $\dfrac{(7)^2+3(7)(9)-10(9)^2}{(7)+5(9)}$

$=\dfrac{[(7)+5(9)][(7)-2(9)]}{(7)+5(9)}$

$=[(7)-2(9)]$

$=7-18$

$=-11$

g) $\dfrac{\left(\frac{2}{7}\right)^2-4\left(\frac{2}{7}\right)\left(\frac{3}{7}\right)-5\left(\frac{3}{7}\right)^2}{\left(\frac{2}{7}\right)-5\left(\frac{3}{7}\right)}$

$=\dfrac{\left[\left(\frac{2}{7}\right)-5\left(\frac{3}{7}\right)\right]\left[\left(\frac{2}{7}\right)+1\left(\frac{3}{7}\right)\right]}{\left(\frac{2}{7}\right)-5\left(\frac{3}{7}\right)}$

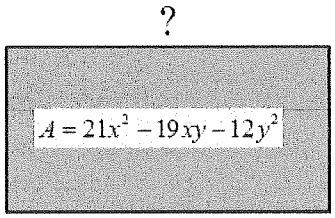
$=\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$

h) $\dfrac{\left(\frac{3}{4}\right)^2+3\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)+2\left(\frac{1}{4}\right)^2}{\left(\frac{3}{4}\right)+2\left(\frac{1}{4}\right)}$

$=\dfrac{\left[\left(\frac{3}{4}\right)+2\left(\frac{1}{4}\right)\right]\left[\left(\frac{3}{4}\right)+1\left(\frac{1}{4}\right)\right]}{\left(\frac{3}{4}\right)+2\left(\frac{1}{4}\right)}$

$=\frac{3}{4}+\frac{1}{4}=1$

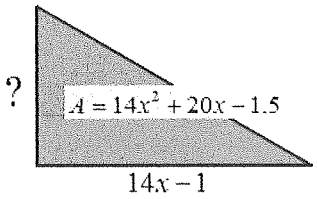
5. The area of a rectangle is given by the expression: $21x^2 - 19xy - 12y^2$ and the width is $7x + 3y$. Find the length of the rectangle:



$21x^2 - 19xy - 12y^2 = (7x+3y)(3x-4y)$

The length of this rectangle is $\boxed{3x - 4y}$.

6. The area of a triangle is given by the expression: $14x^2 + 20x - 1.5$ and the width is $14x - 1$. Find the length of the rectangle:



$2(14x^2+20x-1.5) = 28x^2+40x-3$

$28x^2+40x-3 = (14x-1)(2x+3)$

The length of the triangle is $\boxed{2x+3}$.

7. Suppose the equation $x^2 + kx + 24 = (x-a)(x-b)$, where "a" and "b" are integers. What are all the possible values of "k"?

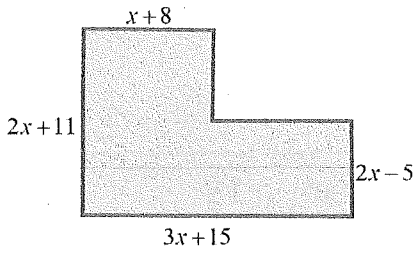
Factors of 24:

1	24	$24+1=25$
2	12	$12+2=14$
3	8	$3+8=11$
4	6	$4+6=10$

$(x-1)(x-24) = x^2-25x+24$
$(x-2)(x-12) = x^2-14x+24$
$(x-3)(x-8) = x^2-11x+24$
$(x-4)(x-6) = x^2-10x+24$

The possible values of k are $\boxed{-25, -14, -11, -10}$.

8. Find the area of the shape in terms of "x"



$(3x+15)(2x+11) - [(3x+15)-(x+8)][(2x+11)-(2x-5)]$

$= 6x^2+63x+165 - (2x+7)(16)$

$= 6x^2+63x+165-(32x+112)$

$= 6x^2+31x+53$

The area of the shape is $\boxed{6x^2+31x+53}$.

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M8H HW CH2 Lesson 8 Multiplying Polynomials

1. Simplify each of the following expressions:

a) $3abc\left(8ab^4c\right)$ $=24a^2b^5c^2$	b) $\left(-13ab^3c\right)\left(-2a^3\right)\left(9bc^3\right)$ $=(26a^4b^3c)(9bc^3)$ $=234a^4b^4c^4$	c) $8ab\left(12-3a+7ac\right)$ $=96ab-24a^2b+56a^2bc$
d) $6abc\left(2a+4b-7\right)$ $=12a^2bc+24ab^2c-42abc$	e) $12ab^3\left(1+2a^2b^3-3a\right)$ $=12ab^3+24a^3b^6-36a^2b^3$	f) $2a^2b\left(15ac^2-bc\right)$ $=30a^3b^3-2a^2b^2c$
g) $-4a^2b\left(2a+4bc\right)$ $=-8a^3b-16a^2b^2c$	h) $4abc^3\left(9a^2-3b^2c\right)$ $=36a^3bc^3-12ab^3c^4$	i) $-3ac\left(3a^3+4a^2b-12c\right)$ $=-9a^4c-12a^3bc+36ac^2$
j) $\dfrac{3x^2y^2-9xy+12x}{3x}$ $=xy^2-3y+4$	k) $\dfrac{3x^2y^2-9xy+12x}{3x}$ $=xy^2-3y+4$	l) $\dfrac{3x^2y^2-9xy+12x}{3x}$ $=xy^2-3y+4$
m) $\dfrac{18ab-36a^3b}{9ab}$ $=2-4a^2$	n) $\dfrac{20ab-16b}{4b}$ $=5a-4$	o) $\dfrac{3ab^2-12ab^3}{3ab}$ $=b-4b^2$
p) $\dfrac{3x^2y^2-9xy+12x}{3x}$ $=xy^2-3y+4$	q) $\dfrac{-24x^3y-16x^2y-12y^3x}{4xy}$ $=-6x^2-4x-3y^2$	r) $\dfrac{-15x^3y^2z-20x^3y+35y^3x^3}{5x^2}$ $=-3xy^2z-4xy+7xy^3$

2. Expand and simplify each of the following

a) $(2x-1)(3x+2)$ $=6x^2+4x-3x-2$ $=6x^2+x-2$	b) $(4x-3)(5x+2)$ $=20x^2+8x-15x-6$ $=20x^2-7x-6$	c) $(2x-3)(5x+4)$ $=10x^2+8x-15x-12$ $=10x^2-7x-12$
d) $(23x+17)(6x-5)$ $=138x^2-115x+102x-85$ $=138x^2-13x-85$	e) $(5x-4)^2$ $=25x^2-20x-20x+16$ $=25x^2-40x+16$	f) $(2x+y-2)^2$ $=4x^2+2xy-4x+2xy+y^2-2y-4x-2y+4$ $=4x^2-8x+4xy-4y+y^2+4$

3. Expand and simplify the following. Combine liketerms and reduce your equation

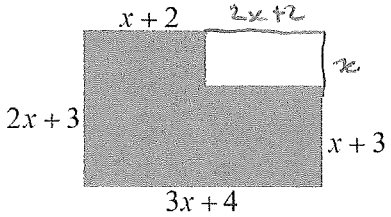
a) $(2x-3)(4x^2-6x+5)$ $=8x^3-12x^2+10x-12x^2+18x-15$ $=8x^3-24x^2+28x-15$	b) $(3x-2)(5x^2+11x-10)$ $=15x^3+33x^2-30x-10x^2-22x+20$ $=15x^3+23x^2-52x+20$
c) $(4x+1)(7x^2-8x+9)$ $=28x^3-32x^2+36x+7x^2-8x+9$ $=28x^3-25x^2+28x+9$	d) $(2x-1)(3x+2)-(5x^2-11x+12)$ $=6x^2+4x-3x-2-5x^2+11x-12$ $=x^2+12x-14$
e) $2x(2x-3)-4x(3-2x)+5x(6-4x)$ $=-2x(3-2x)-4x(3-2x)+10x(3-2x)$ $=4x(3-2x)$ $=-8x^2+12x$	f) $(2x+1)(2x-1)-(3x+2)(3x-2)$ $=4x^2-1-(9x^2-4)$ $=-5x^2+3$
g) $(3x+1)^2-(3x+4)(2x+3)$ $=9x^2+6x+1-(6x^2+17x+12)$ $=3x^2-11x-11$	h) $(2x-1)(5x-3)(4x+2)$ $=(10x^2-11x+3)(4x+2)$ $=40x^3-44x^2+12x+20x^2-22x+6$ $=40x^3-24x^2-10x+6$

i) $(3x-1)^3$ $=(9x^2-6x+1)(3x-1)$ $=27x^3-18x^2+3x-9x^2+6x-1$ $=27x^3-27x^2+9x-1$	J) $(3x+1)(5x+2)(3x-4)-2x^3+4x^2$ $=(15x^2+11x+2)(3x-4)-2x^3+4x^2$ $=45x^3+33x^2+6x-60x^2-44x-8-2x^3+4x^2$ $=43x^3-23x^2-38x-8$
k) $(2x-1)(2x+1)(2x-1)(2x+1)$ $=(4x^2-1)(4x^2-1)$ $=16x^4-8x^2+1$	L) $(3x+1)^2-(x-4)^2-(2x+3)^2$ $=(9x^2+6x+1)-(x^2-8x+16)-(4x^2+12x+9)$ $=4x^2+2x-24$

4. What is the difference between "simplifying" an expression and "solving" an equation?

Simplifying an expression makes it neat and easy to read.
Solving an equation means finding the value of x.

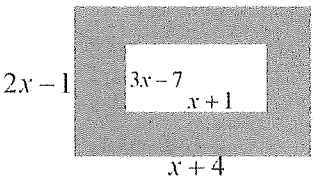
5. Find the area of the shaded region for each of the following diagrams:



$(2x+3)(3x+4)-(x)(2x+2)$

$=6x^2+17x+12-(2x^2+2x)$

$=4x^2+15x+12$

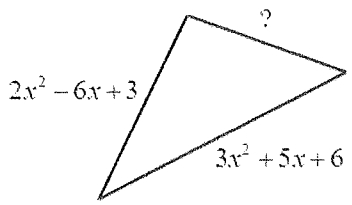


$(2x-1)(x+4)-(3x-7)(x+1)$

$=(2x^2+7x-4)-(3x^2-4x-7)$

$=-x^2+11x+3$

6. The perimeter of a triangle is $6x^2+7x-12$. Find the missing side.



$6x^2+7x-12-(2x^2-6x+3+3x^2+5x+6)$

$=6x^2+7x-12-(5x^2-x+9)$

$=x^2+8x-21$

The missing side is $\boxed{x^2+8x-21}$.

7. Given the area and length of one side of a rectangle, find the width in terms of "x":

a) $A=10x^2-11x-6,\ L=(2x-3)$

$10x^2-11x-6=(2x-3)(5x+2)$

The width is $\boxed{5x+2}$.

b. $A=18x^2-93x+84;\ L=(6x-7)$

$18x^2-93x+84=(6x-7)(3x-12)$

The width is $\boxed{3x-12}$.

8. Given that : $(1+x+x^2+x^3+x^4)^2=Ax^8+Bx^7+Cx^6+Dx^5+Ex^4+Fx^3+Gx^2+Hx+1$

What is the value of $A+B+C+D+E+F+G+H$?

$(x^4+x^3+x^2+x+1)(x^4+x^3+x^2+x+1)$

$=x^8+x^7+x^6+x^5+x^4+x^7+x^6+x^5+x^4+x^3+x^6+x^5+x^4+x^3+x^2+x^5+x^4+x^3+x^2+x+x^4+x^3+x^2+x+1$

$=x^8+2x^7+3x^6+4x^5+5x^4+4x^3+3x^2+2x+1$

$1+2+3+4+5+4+3+2$

$=24$

The value of $A+B+C+D+E+F+G+H$ is $\boxed{24}$.

Name:_____ Date:_____

M8H HW Ch2 Lesson 9 Factoring a Difference of Squares

1. Multiply each of the following binomials with it's conjugate:

a) $(2x-3)(2x+3)$ $=4x^2-9$	b) $(4x-3)(4x+3)$ $=16x^2-9$
c) $(7x-1)(7x+1)$ $=49x^2-1$	d) $(9x-5)(9x+5)$ $=81x^2-25$
e) $(10x-11y)(10x+11y)$ $=100x^2-121y^2$	f) $(15x+4y)(15x-4y)$ $=225x^2-16y^2$

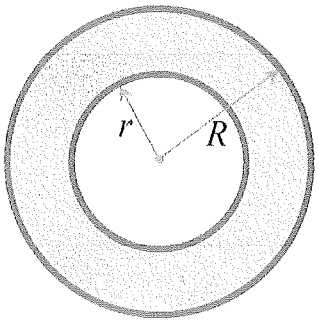
2. Indicate the missing value in the box for each of the following

a) $16-25x^2=(4-\boxed{?})(4+\boxed{?})$ $16-25x^2=(4-5x)(4+5x)$	b) $121-81x^2=(11-\boxed{?})(\boxed{?}+\boxed{?})$ $121-81x^2=(11-9x)(11+9x)$
c) $\boxed{?}-100y^2=(5x-\boxed{?})(5x+\boxed{?})$ $25x^2-100y^2=(5x-10y)(5x+10y)$	d) $98y^2-162x^2=\boxed{?}(\boxed{?}-\boxed{?})(\boxed{?}+\boxed{?})$ $98y^2-162x^2=2(7y-9x)(7y+9x)$
e) $\boxed{?}-144x^4=4(9x-\boxed{?})(\boxed{?}+\boxed{?})$ $324x^4-144x^2=4(9x-6x^2)(9x+6x^2)$	f) $\boxed{?}-75x^6y=3y(4-\boxed{?})(4+\boxed{?})$ $48y-75x^6y=3y(4-5x^3)(4+5x^3)$
g) $45x^6y-\boxed{?}=5y(\boxed{?}-7y^3z)(\boxed{?}+\boxed{?})$ $45x^6y-245y^7z^2=5y(3x^3-7y^3z)(3x^3+7y^3z)$	h) $300x^4y^2-\boxed{?}=3y^2(\boxed{?}-7z)(\boxed{?}+\boxed{?})$ $300x^4y^2-147y^3z^2=3y^2(10x^2-7z)(10x^2+7z)$

3. Factor each of the following expressions:

a) $9x^2-4y^2$ $=(3x+2y)(3x-2y)$	b) $81a^2-25b^2$ $=(9a-5b)(9a+5b)$	c) $16x^2-49y^2$ $=(4x-7y)(4x+7y)$
d) $\frac{x^2}{16}-\frac{y^2}{49}$ $=\left[\frac{x}{4}-\frac{y}{7}\right]\left[\frac{x}{4}+\frac{y}{7}\right]$ $=\left(\frac{7x-4y}{28}\right)\left(\frac{7x+4y}{28}\right)$	e) $27x^3-12xy^2$ $=3x(9x^2-4y^2)$ $=3x(3x-2y)(3x+2y)$	f) $50a^2b-98b^3$ $=2b(25a^2-49b^2)$ $=2b(5a-7b)(5a+7b)$
g) $(2x-1)^2-9x^2$ $=[(2x-1)-3x][(2x-1)+3x]$ $=(-x-1)(5x-1)$	h) $(2x-1)^2-(3x+2)^2$ $=[(2x-1)-(3x+2)][(2x-1)+(3x+2)]$ $=(-x-3)(5x+1)$	i) $(4x+3)^2-(x+5)^2$ $=[(4x+3)-(x+5)][(4x+3)+(x+5)]$ $=(3x-2)(5x+8)$
J) $4(2x-y)^2-25z^2$ $=[2(2x-y)-5z][2(2x-y)+5z]$ $=(4x-2y-5z)(4x-2y+5z)$	k) $18x^2y^2-50y^4$ $=2y^2(9x^2-25y^2)$ $=2y^2(3x-5y)(3x+5y)$	L) $81a^2-(3a+2b)^2$ $=[9a-(3a+2b)][9a+(3a+2b)]$ $=(6a-2b)(12a+2b)$
m) $5x^4-80$ $=5(x^4-16)$ $=5(x^2-4)(x^2+4)$ $=5(x-2)(x+2)(x^2+4)$	n) $(4x^2-4)^2-81x^4$ $=[(4x^2-4)-9x^2][(4x^2-4)+9x^2]$ $=[-5x^2-4][13x^2-4]$	0) $121x^2+36y^2$ Cannot be factored

4. Two concentric circles are overlapping each other as shown in the diagram. Which of the following equations below provides the area between the two circles?



$eq\#1: A = (r+R)^2 \times \pi$

$eq\#2: A = (r+R)(r-R)\pi$

$eq\#3: A = (R+r) \times \pi^2$

$eq\#4: A = (R+r)(R-r) \times \pi$

$eq\#5: A = (R^2 + r^2) \times \pi$

$R^2\pi - r^2\pi = \pi(R^2 - r^2)$

$= \pi(R-r)(R+r)$

∴ eq #2 is correct

5. What is the value of $1000^2 - 999^2 = (\boxed{1000} + \boxed{999})(\boxed{1000} - \boxed{999})$?

$= (1999)(1)$

$= 1999$

The value of $1000^2 - 999^2$ is $\boxed{1999}$.

6. The difference of two numbers is 20, and the difference of their squares is 800. What is the sum of the two numbers?

$(x+20)^2 - x^2 = [(x+20) - x][(x+20) + x] = 800$

$800 = (20)(2x+20)$

$800 = 40x + 400$

$x = 10$

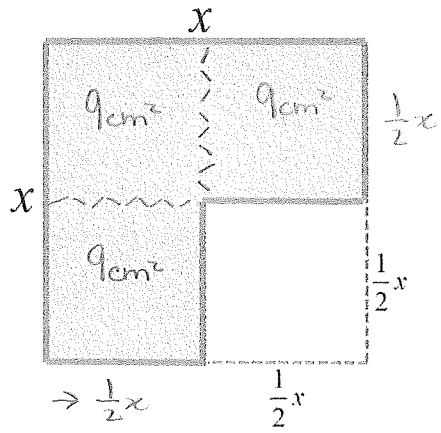
$30^2 - 10^2 = (30-10)(30+10)$

$= (20)(40)$

$= 800$

The sum of the two numbers is $\boxed{40}$.

7. A square with a quarter of it cut out is shown below. If the remaining area is $27cm^2$, then what is the value of "x"?



①

$x^2 - \left(\frac{x}{2}\right)^2 = \left[x - \frac{x}{2}\right]\left[x + \frac{x}{2}\right] = 27$

$27 = \left(\frac{x}{2}\right)\left(\frac{3x}{2}\right)$

$27 = \frac{3x^2}{4} = 3 \cdot \frac{x^2}{4}$

$9 = \frac{x^2}{4}$

$x = 6$

②

$\left(\frac{x}{2}\right)^2 = 9$

$\frac{x^2}{4} = 9$

$x^2 = 36$

$x = 6$

The value of x is $\boxed{6}$.

8. The positive difference of two perfect squares is 32. What are the possible values of these two perfect squares?

32 =
~~1 · 32~~
2 · 16
4 · 8

① $x-y=2$ $x+y=16$
$x=y+2$ $2y+2=16$
$y=7$
$x=9$ $y=7$
$81-49=32$

② $x-y=4$ $x+y=8$
$x=y+4$ $2y+4=8$
$y=2$
$x=6$ $y=2$
$36-4=32$

9. What is $1{,}000{,}000^2 - 999{,}999^2$? Do not use a calculator.

$1000000^2-999,999^2$
$=(1000000-999999)(1000000+999999)$
$=1(1999999)$
$=1999999$

$1{,}000{,}000^2-999{,}999^2$ is 1999999.

10. The number 2001 can be written as a difference of squares, $x^2 - y^2$ where "x" and "y" are positive integers in four different ways. What are the four possible ways?

2001
3 · 667
23 · 29
1 · 2001
3 · 667
69 · 29
87 · 23

① $(x-y)=3$ $x+y=667$
$x=y+3$ $2y+3=667$
$2y=664$
$y=332$
$x=335$ $y=332$
$335^2-332^2=2001$

② $x-y=23$ $x+y=87$
$x=23+y$ $2y+23=87$
$x=55$ $y=32$
$55^2-32^2=2001$

③ $x-y=29$ $xy=69$
$x=29+y$ $2y+29=69$
$y=20$
$x=49$
$49^2-20^2=2001$

④ $x-y=1$ $x+y=2001$
$x=y+1$ $2y+1=2001$
$y=1000$
$x=1001$
$1001^2-1000^2=2001$

11. Two numbers are such that their difference, their sum and their product are to one another as 1 : 7 : 18. The product of the two numbers are:

a) 6 b) 12 c) 24 d) 48 e) none of these

27

$\frac{a-b}{a+b}=\frac{1}{7}$ $\frac{a-b}{a\cdot b}=\frac{1}{18}$ $\frac{a+b}{a\cdot b}=\frac{7}{18}$

$7a-7b=a+b$ $18a-18b=a\cdot b$ $18a+18b=7ab$

$6a=8b$ $36a=8ab$ $b=\frac{9}{2}$ $a=\frac{-9}{3}\cdot\frac{y^2}{3y}$

$a=\frac{4}{3}b$ $8b=8b$ $a=6$

$6-\frac{9}{2}=\frac{3}{2}$
$6+\frac{9}{2}=\frac{21}{2}$
$6\cdot\frac{9}{2}=27$

$1\cdot\frac{3}{2}=\frac{3}{2}$
$7\cdot\frac{3}{2}=\frac{21}{2}$
$18\cdot\frac{3}{2}=27$

12. The number 2005 can be written in the form of $a^2 - b^2$, where "a" and "b" are positive integers less than 1000 in exactly one way. What is the value of $a^2 + b^2$?

2005
5 · 401

$(a-b)=5$ $(a+b)=401$
$a=b+5$ $[(b+5)+b]=401$
$2b+5=401$
$2b=396$
$b=198$
$a=203$

203^2+198^2
$=41209+39204$
$=80413$

The value of a^2+b^2 is 80413.

Name: Francis Date:

M8H HW CH2 Lesson 10: Factoring and Solving Trinomials

1. Given each pair of binomials, expand and simplify:

a. $(x-3)(x+4)$

$=x^2+4x-3x-12$

$=x^2+x-12$

b. $(x+11)(x-9)$

$=x^2-9x+11x-99$

$=x^2+2x-99$

c. $(2x+3)(3x-1)$

$=6x^2-2x+9x-3$

$=6x^2+7x-3$

e. $(7x-3)(4x+2)$

$=28x^2+14x-12x-6$

$=28x^2+2x-6$

f. $(10x-3)(4x-2)$

$=40x^2-20x-12x+6$

$=40x^2-32x+6$

g. $(8x-3)(3x-8)$

$=24x^2-64x-9x+24$

$=24x^2-73x+24$

2. Given each pair of binomials, solve for "x":

a. $(x+9)(x+21)=0$

$x=-9 \quad x=-21$

b. $4(x-3)(x+3)=0$

$x=3 \quad x=-3$

c. $(x+81)(x-29)=0$

$x=-81 \quad x=29$

e. $(2x-5)\left(x-\frac{1}{2}\right)=0$

$x=\frac{5}{2} \quad x=\frac{1}{2}$

f. $x(3x+1)=0$

$x=0 \quad x=\frac{1}{3}$

g. $2(5-2x)\left(\frac{1}{3}-x\right)=0$

$x=\frac{5}{2} \quad x=\frac{1}{3}$

3. Given each expression, find the missing value in the box:

a. $x^2-11x-12=\left(x-\boxed{?}\right)(x+1)$

$x^2-11x-12=(x-12)(x+1)$

b. $x^2-29x+120=\left(x-\boxed{?}\right)(x-5)$

$1 \quad -5$

$1 \quad -24$

$x^2-29x+120=(x-24)(x-5)$

c. $5x^2+6x+1=\left(5x+\boxed{?}\right)(x+1)$

$5x^2+6x+1=(5x+1)(x+1)$

d. $2x^2-23x+11=\left(2x-\boxed{?}\right)(x-11)$

$2 \quad -1$

$1 \quad -11$

$2x^2-23x+11=(2x-1)(x-11)$

4. Factor each of the following expressions. Show all your steps and work:

a. x^2+7x+6

$1-6 \quad 6+1=7$

$1-1$

$=(x+6)(x+1)$

b. $x^2+25x+24$

$1-24 \quad 24+1=25$

$1-1$

$=(x+24)(x+1)$

c. $x^2+10x+21$

$1-3 \quad 3+7=10$

$1-7$

$=(x+3)(x+7)$

d. $x^2-10x+24$

$1-6 \quad -6-4=-10$

$1-4$

$=(x-6)(x-4)$

e. $x^2+3x-40$

$1-8 \quad 8-5=3$

$1--5$

$=(x+8)(x-5)$

f. $4x^2+9x+2$

$4-2\to 8 \quad 8+1=9$

$1-1$

$=(4x+1)(x+2)$

g. $2x^2+5x+2$

$2-2\to 4 \quad 4+1=5$

$1-1$

$=(2x+1)(x+2)$

h. $2x^2-11x+15$

$2\times -5 \quad -6-5=-11$

$1\times -3\to -6$

$=(2x-5)(x-3)$

i) $21x^2+17x-30$

$3\times 5\to 35 \quad 35-18=17$

$7\times -6\to -18$

$=(7x-6)(3x+5)$

j) $2x^2-7x+5$

$1\times -1 \quad -2-5=-7$

2×-5

$=(2x-5)(x-1)$

k) $5x^2-13x-6$

$1-2 \quad -15+2=-13$

$5-3\to -15$

$=(5x+2)(x-3)$

L) $7x^2+9x-10$

$1\times 2\to 14 \quad 14-5=9$

7×-5

$=(7x-5)(x+2)$

m) $21+26x-15x^2$

$3--3\to 9 \quad 35-9=26$

$7-5\to 35$

$=(3x+5)(7x-3)$

n) $2x^2-9xy-45y^2$

$1-15 \quad -15+6=-9$

$2-3\to 6$

$=(x+3y)(2x-15y)$

o) $5x^4-9x^2-2$

$1\times -2\to -10 \quad -10+1=-9$

5×1

$=(5x^2+1)(x^2-2)$

p) $6-7x^2+2x^4$

$2\times -3 \quad -4-3=-7$

$1\times -2\to -4$

$=(2x^2-3)(x^2-2)$

5. Factor each of the following expressions and solve for "x". Show all your steps and work:

a. $x^2+8x+12=0$

$1\times 6 \quad 6+2=8$

1×2

$(x+6)(x+2)=0$

$x=-6 \quad x=-2$

b. $x^2+17x+72=0$

$1-8 \quad 8+9=17$

$1-9$

$(x+8)(x+9)=0$

$x=-8 \quad x=-9$

c. $x^2+2x-15=0$

$1\times 5 \quad 5-3=2$

1×-3

$(x+5)(x-3)=0$

$x=-5 \quad x=3$

d. $x^2-7x-170=0$

$1-17 \quad -17+10=-7$

$1-10$

$(x+10)(x-17)=0$

$x=-10 \quad x=17$

e. $x^2 - 64 = 0$

$64 - 64 = 0$

$x^2 = 64$

$x = 8$

f. $100 - x^2 = 0$

$100 - 100 = 0$

$x^2 = 100$

$x = 10$

g. $(2x-1)^2 - 16 = 0$

$(4x^2 - 4x + 1) - 16 = 0$

$4x^2 - 4x - 15 = 0$

$2 \quad -5 \to -10$

$2 \quad 3 \to 6$

$-10 + 6 = -4$

$(2x+3)(2x-5) = 0$

$x = -\frac{3}{2} \quad x = \frac{5}{2}$

h. $2x^2 - 11x + 15 = 0$

$1 \quad -3 \to -6$

$2 \quad -5$ $\quad -6 - 5 = -11$

$(x-5)(2x-3) = 0$

$x = 5 \quad x = \frac{3}{2}$

i) $13x^2 + 8x - 5 = 0$

$1 \quad 1$

$13 \quad -5$ $\quad 13 - 5 = 8$

$(13x+1)(x-5) = 0$

$x = \frac{1}{13} \quad x = 5$

j) $2x^2 - 25x - 13 = 0$

$2 \quad 1$

$1 \quad -13 \to -26$

$-26 + 1 = -25$

$(2x+1)(x-13) = 0$

$x = -\frac{1}{2} \quad x = 13$

k) $2x^2 - 7x + 6 = 0$

$1 \quad -3$

$2 \quad -2 \to -4$

$-4 - 3 = -7$

$(2x-3)(x-2) = 0$

$x = \frac{3}{2} \quad x = 2$

L) $10x^2 + 49x + 49 = 0$

$2 \quad 7 \to 14$

$5 \quad 7 \to 35$ $\quad 14 + 35 = 49$

$(2x+7)(5x+7) = 0$

$x = -\frac{7}{2} \quad x = -\frac{7}{5}$

m) $(x+2)^2 + 8(x+2) - 20 = 0$

$1 \quad 10$

$1 \quad -2$ $\quad 10 - 2 = 8$

$[(x+2)+10][(x+2)-2] = 0$

$(x+12)(x) = 0$

$x = -12 \quad x = 0$

n) $(x-3)^2 + 10(x-3) + 9 = 0$

$1 \quad -1$

$1 \quad -9$ $\quad 9 + 1 = 10$

$[(x-3)+1][(x-3)+9] = 0$

$(x-2)(x+6) = 0$

$x = 2 \quad x = -6$

o) $2(x+1)^2 - (x+1) - 6 = 0$

$2 \quad 3$

$1 \quad -2 \to -4$ $\quad -4 + 3 = -1$

$[(x+1)-2][2(x+1)+3] = 0$

$(x-1)(2x+5) = 0$

$x = 1 \quad x = -\frac{5}{2}$

p) $4(x+2)^2 = 6 - 5(x+2)$

$4(x+2)^2 + 5(x+2) - 6 = 0$

$[4(x+2)-3][(x+2)+2] = 0$

$(4x+5)(x+4) = 0$

$x = -\frac{5}{4} \quad x = -4$

o) $x^4 - 256 = 0$

$x^4 = 256$

$x = 4$

q) $x^4 = 10 - 9x^2$

$x^4 + 9x^2 - 10 = 0$

$(x^2+10)(x^2-1) = 0$

$x = 1$

r) $r^4 - 17r^2 + 16 = 0$

$1 \quad -16$

$1 \quad -1$ $\quad -16 - 1 = -17$

$(r^2-16)(r^2-1) = 0$

$r = 4 \quad r = 1$

s) $x^4 - 29x^2 + 100 = 0$

$1 \quad -25$

$1 \quad -4$ $\quad -25 - 4 = -29$

$(x-25)(x-4) = 0$

$x = 25 \quad x = 4$

t) $4(x^2 - 6x + 9)^2 - 12(x^2 - 6x + 9) = -9$

$4(x^2-6x+9)^2 - 12(x^2-6x+9) + 9 = 0$

$2 \quad -3$

$2 \quad -3$ $\quad -6 - 6 = -12$

$[2(x^2-6x+9)-3][2(x^2-6x+9)-3] = 0$

$[2(x^2-6x+9)-3] = 0$

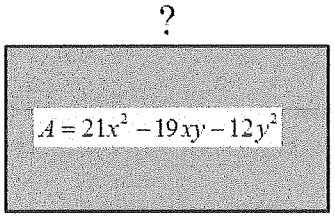
$(2x^2 - 12x + 15) = 0$

$2 \quad 3$

$1 \quad 5$

?

6. The area of a rectangle is given by the expression: $21x^2 - 19xy - 12y^2$ and the width is $7x + 3y$. Find the length of the rectangle:



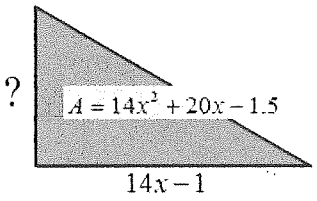
$21x^2 - 19xy - 12y^2$

$7 \times 3 \to 9$, $3 \times -4 \to -28$, $-28 + 9 = -19$

$= (7x + 3y)(3x - 4y)$

The length of the rectangle is $\boxed{3x - 4y}$.

7. The area of a triangle is given by the expression: $14x^2 + 20x - 1.5$ and the width is $14x - 1$. Find the length of the rectangle:



$2(14x^2 + 20x - 1.5)$

$= 28x^2 + 40x - 3$

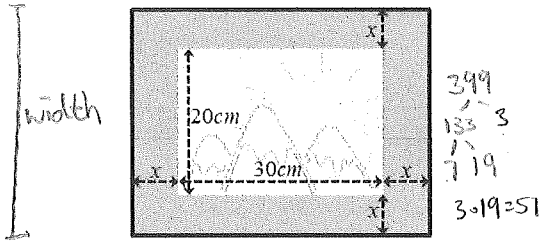
$14 \to -3 \to -42$, $-42 - 2 = -40$

$2 \to -1 \to -2$

$= (14x - 1)(2x + 3)$

The width of the triangle is $\boxed{2x + 3}$.

8. A photograph that is 20cm by 30cm is framed with a uniform mat board as shown below. If the area of the photo with the mat is 999cm^2, then what is the width of the mat?



$399 = 3 \times 133 = 3 \times 7 \times 19$

$3 \cdot 19 = 57$

$(2x + 20)(2x + 30) = 999$

$4x^2 + 100x + 600 - 999 = 0$

$4x^2 + 100x - 399 = 0$

$2 \to 57 \to 114$, $2 \to -7 \to -14$, $114 - 14 = 100$

$(2x - 7)(2x + 57) = 0$

$x = \frac{7}{2}$ ~~$x = -\frac{57}{2}$~~ Cannot be negative

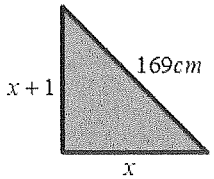
$(7 + 20)(7 + 30)$

$= 27 \cdot 37$

$= 999$

The width of the mat is $\boxed{27}$ cm.

9. Find the length of the base for the following triangle:



$(x + 1)^2 + x^2 = 169^2$

$x^2 + 2x + 1 + x^2 - 169^2 = 0$

$2x^2 + 2x + 1 - 28561 = 0$

$2x^2 + 2x - 28560 = 0$

1×120, 2×-238, $240 - 238 = 2$

$(x + 120)(2x - 238) = 0$

~~$x = -120$~~ Cannot be negative $\boxed{x = 119}$

$119^2 + 120^2$

$= 14161 + 14400$

$= 28561 = 169^2$

The length of the base of the triangle is $\boxed{119}$ cm.

$28560 = 2^4 \cdot 3 \cdot 5 \cdot 7 \cdot 17$

1. 28560
2. 14280
3. 9520
4. 7140
5. 5712
6. 4760
7. 4080
8. 3570
10. 2856
12. 2380
120. 238